CHAPTER 7

GENERAL EQUILIBRIUM EFFECTS OF MORE INEQUALITY OR SUPERVISION COSTS

The "supply-siders" claim more inequality is good for the U.S. economy. More inequality clearly isn't good for the simple Clone economy of Chapter 1.

Even without transactions costs, more inequality lowers total labor supply and output of the Clone economy, though it raises the economywide wage! With transactions costs, more inequality lowers total labor supply and output even more. It raises wages of richer people, drives down wages of poorer people, and lowers average wages for the whole economy. Perversely, more inequality raises economywide output per manhour, by reducing employment proportionally more than output. (So much for the shibboleth of high labor productivity!)

Higher transactions costs, given inequality, have much the same effect as more inequality given transactions costs.

Sec. 7.1 summarizes basic results of Chp. 7. Sec. 7.2 draws some further implications for land use patterns, and communist revolutions.

7.1 Summary[A]

Suppose for convenience there are only two farmers in the Clone economy of Chp. 1. Call the farmer with half or more land the "landlord" and the other farmer the "peasant". When distribution is unequal, the landlord may hire the peasant, subject to a supervision requirement, with neither acting as monopolist or monopsonist.

What happens as distribution of land between the two becomes more unequal?

What happens if the required rate of supervision increases?

Table 7.1 summarizes the overall results as distribution of land goes from equality to complete inequality between landlord and peasant, and as transactions costs rise, given inequality.

As inequality increases, the economy goes through three phases, as explained in Sec. 7.3:

In Phase I, the supervision requirement precludes the landlord from hiring the peasant, so the two behave as independent landowners of different size. Sec. 7.4 describes Phase I.

In Phase II, the landlord hires and supervises the peasant, but continues to perform the same work alongside.

In Phase III, the landlord merely supervises his employee.

What happens within the phases depends on the required supervision rate.

Greater Inequality at a Zero Supervision Rate (Sec. 7.5):

At a zero supervision rate, Phase I does not exist. The moment distribution becomes unequal, the landlord hires the peasant and works beside him at a wage initially equal to the marginal product of labor on both pieces of land at equal distribution.

As distribution becomes more unequal, income effect leads the landlord's personal labor supply to fall faster than the peasant's hired labor supply rises at a given wage. So the wage rises to equate supply and demand for hired labor. As the wage rises, total labor supply and output fall.

The economy goes into Phase III when the landlord stops doing any work himself. Now the peasant supplies all the labor of cultivation on both his own and the landlord's land. As the peasant's own land decreases, income effect leads him to offer more hired labor at the

Table 7.1

Effect of Increased Inequality with Supervision Cost: A

Effect of Increased Supervision Cost, Given Inequality: B

	A		B	
	lo*	hi	lo	hi
1. Labor:				
Peasant's total personal:	+	-	-	
Self = applied:	-		+	
Hired:	+		-	
Landlord's total personal:	-	+	+	+?
Self:	-		+	
Supervisory:	+		+	-?
Applied = self + hired:	+		-	
Total; personal, applied:	-		-	
2. Applied labor per acre:				
On peasant's land:	-	+	+	
On landlord's land:	-		-	
Simple average:	-	+	+	
Overall (weighted average).	-		-	
3. Wage and MP labor (MPL):				
Peasant:	+	-	-	
Landlord; wage:	+		+	?
MPL:	+		+	
Difference; wage:	+		+	?
MPL:	+		+	
Simple avg; wage:	+	-	-	
MPL:	+	-	-	
Weigtd avg:	+		+	
4. Output:				
On peasant's land:	-		+	
On landlord's land:	+		-	
Total:	-		-	
5. MP land = rent:				
On peasant's land:	-	+	+	
On landlord's land:	-		-	
Simple avg:	-	+	+	
Weighted avg:	-		-	

* "lo": "low supervision rate". "hi": "high supervision rate".

Table 7.1, cont'd

	A		B	
	lo	hi	lo	hi
6. Output/manhour:				
On peasant's land:	+	-	-	
On landlord's land:		+	+	
Overall:		+	+	
7. Output/acre:				
On peasant's land:	-	+	+	
On landlord's land:		-	-	
Overall:		-	-	
8. Labor share of output (labor cost/output):				
On peasant's land:	+	-	-	
On landlord's land:		+	+	
Overall:		+	+	
9. Ordinary income:				
Peasant's:		-	-	
Landlord's:		+	?	
Total:		-	-	

given wage. So the wage falls again to equate supply and demand for hired labor, and total labor supply and output rise again. However, total labor supply does not rise back to its level at equal distribution, and the peasant's wage does not fall back to its value at equal distribution. So total production does not rise back to its equal distribution level either.

Section 7.5 develops in detail the effect of greater inequality in the absence of a supervision requirement. Table 7.2 in Section 7.5 summarizes the results.

Greater Inequality, Given a Supervision Requirement (Sec. 7.6):

Given a supervision requirement, and hence a supervision cost, the landlord does not hire the peasant the moment distribution becomes unequal.

First, Phase I applies. In Phase I, by the assumption that labor supply is a concave function of land size, total labor supply falls. For as distribution becomes more unequal, the peasant's labor supply on his shrinking piece of land falls faster than the landlord's labor supply increases. So total output falls.

The ratio of labor to land rises on the peasant's land, so that the peasant's wage, which equals his marginal product of labor, falls. The ratio falls on the landlord's land, so that the landlord's wage and marginal product of labor rise. Section 7.4 describes Phase I in detail. Table 7.3, column 1, in Section 7.6 summarizes effects of increased inequality in Phase I.

The landlord hires the peasant only when distribution has become sufficiently unequal that the marginal product of labor on the peasant's land equals the marginal product of labor on the landlord's land less supervision costs. Phase II begins here.

At the start of Phase II, the landlord pays the peasant a wage equal to the marginal product of labor on the peasant's land. As distribution becomes more unequal, this wage falls at first, then rises again. However, total labor falls continuously. The higher the supervision rate, the larger the region of distribution within Phase II in which the wage falls, and the less the net increase (if any) in wage during Phase II. Also, the higher the rate, the larger the decrease in total labor supply, and hence output.

The economy goes into Phase III when the landlord ceases to cultivate the land, but only supervises the peasant. Even for a zero supervision rate, increased inequality leads the peasant to offer more hired labor than the landlord demands, so the wage falls. However total labor supply and output rise. But the higher the supervision rate, the lower the landlord's demand for the peasant's hired labor, and the faster the wage falls. For a high enough supervision rate, total labor supply and output fall.

In Phase II, the landlord's wage rises or falls in proportion to the peasant's wage--since the landlord performs identical labor. In Phase III, the landlord's wage simply rises.

At a very low supervision rate, the economy behaves mostly like the economy with no supervision rate. At a high supervision rate, the economy behaves pretty much as in Phase I: total labor supply, output, and the peasant's wage decline continually as distribution becomes more unequal.

Section 7.6 develops in detail the effects of greater inequality, given a low or high supervision rate. Table 7.3, column 2, in Section 7.6, summarizes the effects in Phase II. Column 3 summarizes the effect

of greater inequality in Phase III. Column 4 summarizes the combined effect of greater inequality given a low or high supervision requirement.

Section 7.7 develops the effects of a higher supervision rate, given inequality. Table 7.3, Section 7.7, summarizes the results. Column 1 shows Phase II, column 2 shows Phase III, and column 3 shows the combined effect.

7.2 Further Implications[A]

1. The results of Chp. 7 have implications for land use patterns in an economy where land varies in quality: If greater inequality lowers the wage of poorer people, then it makes previously submarginal land supramarginal for any economic activity. In the classic location theory model of Chp. 3, greater inequality spreads out the bullseye of activities, pushing the boundaries between activities further from the center. Consequently, the more unequal the distribution of wealth, the greater the area and the lower the average quality of land a particular activity occupies. So paradoxically, greater inequality simultaneously reduces output, and increases the area of land in production.

As an example, in parts of Latin America large haciendas run low intensity cattle operations on the fertile valley bottoms--quite visibly pushing peasant farming out onto what should be submarginal land: the steep eroding hillsides.

2. Western economists love to point out the hideous inefficiencies of state planning in the communist countries. Chp. 7 suggests why, despite these inefficiencies, communist revolutions (not conquests) have to varying degrees succeeded in generating economic growth and raising average standards of living in previously very poor and stagnant areas, like the Soviet Union, China, or Cuba. For prior to revolutions, these

countries suffered from extreme inequality and archaic, corrupt government, which created a high level of transactions costs. According to Chp. 7, such inequality and transactions costs make an economy very inefficient. So communist revolutions, with their strong redistributive policies, presumably brought a net reduction in economic inefficiency.

7.3 Distribution and Boundary Conditions[C]

Chapter 1 presented simple models of a landowner under four different possible circumstances:

a. The landowner performs all the labor on his own land. (Sec. 1.5).

b. The landowner performs all the labor on his own land and works for hire elsewhere at a given wage. (Sec. 1.6).

c. The landowner works on his own land and hires additional labor at a given wage, which he must supervise at a given rate. (Sec. 1.7).

d. The landowner does not work directly on his own land, but only hires and supervises labor. (Sec. 1.8).

These are all partial equilibrium models. They show changes in behavior of the landowner and his firm if land size, or given wage, or supervision rate increase, holding the other variables constant.

However, the models easily combine into general equilibrium models, showing what happens as the distribution of a fixed quantity of land becomes more unequal, or the required supervision rate for the economy increases. In these general equilibrium models, the wage for employees no longer remains given, but depends on the distribution of land and the required supervision rate.

Assumptions About Distribution

The general equilibrium models presented in this chapter assume a two-person economy. Two landowners share a fixed quantity of land, B. The "landlord", denoted by superscript "d", owns half or more of the land. The "peasant", denoted by superscript "p", owns half or less. So if the landlord owns T acres, $T \geq B/2$, the peasant owns B-T acres, $B-T \leq B/2$. So $T^d = T$, and $T^P = B-T$. However, this landlord and peasant still behave as they would in a perfectly competitive

economy, neither acting as monopolist or monopsonist.

However, sometimes it helps to consider a model with many landowners, divided into two groups, landlords and peasants.

In that case, imagine a population of M persons, occupying the total land area, B. N of these persons are landlords. They occupy a portion T of the land, each owning a piece of size T/N. There are M-N peasants, occupying B-T of the land, each owning a piece of size (B-T)/(M-N). The landlords' pieces are greater than or equal to the peasants' pieces: $T/N \geq (B-T)/(M-N)$. This formulation means that distribution can vary from complete equality, where $T/N = (B-T)/(M-N)$, -- to the point that one landlord owns all the land and the rest of the population owns nothing: $N = 1$ and $T = B$.

The larger the population, M, the more extreme inequality can become. For example, with a population of two, distribution can range from equality to 50% of the population owning 100% of the land. But with a population of 100, distribution can range from equality to 1% of the population owning 100% of the land.

However, a decrease in the proportion of landlords in the population produces the same sort of effects as an increase in the proportion of land belonging to a given number of landlords. So for most purposes it suffices to change the proportion of land between two landowners.

Boundary Conditions and the Three Phases of Distribution

Obviously when two otherwise identical landowners also own the same quantity of land, neither of them will hire the other's labor. However, as distribution becomes unequal and more unequal, there may come a point where the landowner with more land, the "landlord", begins to hire

labor from the landowner with less land, the "peasant". The Kuhn-Tucker conditions for the landowner who works for hire (Sec. 1.6), and for the landowner who hires but works his own land (Sec. 1.7) -- give the distribution at which the peasant starts to work for the landlord. These are the conditions, with those from Sec. 1.6 applying to the peasant, and those from 1.7 applying to the landlord:

$$(1.6.6) \quad w^P - f_2^P = 0$$

$$(1.6.7) \quad w^P - v \geq 0 \qquad (w^P - v)H = 0$$

$$(1.7.6) \quad w^d - f_2^d \geq 0 \qquad (w^d - f_2^d)S^d = 0$$

$$(1.7.7) \quad v + kw^d - f_2^d \geq 0 \qquad (v + kw^d - f_2^d)H = 0$$

If the landlord hires the peasant ($H > 0$), then v is the "market" wage. However, if the inequalities hold in (1.6.7) and (1.7.7), then the equality holds in (1.7.6). $S^d = L^d$ -- the landlord supplies all his own labor on his land, and the landowner model of Sec. 1.5 applies to both peasant and landlord. So the peasant's and landlord's labor, L^P and L^d, depend solely on their respective quantities of land, B-T and T. Then a little rearranging of the Kuhn-Tucker conditions above shows that the marginal product of the peasant's labor on his own land exceeds the marginal product of his labor on the landlord's land, after deducting supervision costs. Consequently, there exists no value of v high enough to induce the peasant to work for hire, yet low enough to induce the landlord to hire him:

$$(3.1) \quad w^P = f_2(B-T,L^P) > (1-k)f_2(T,L^d) = (1-k)w^d$$

As distribution becomes more unequal, the marginal product of labor

on the peasant's land falls, and the marginal product of labor on the landlord's land rises. Eventually, the equalities hold in (1.6.7) and (1.7.7). The landlord just begins to hire the peasant, so H = 0. The landlord now owns T* acres, and the peasant owns B-T*. There is now a "market" wage v* such that:

$$(3.2) \qquad w^P = f_2(B-T^*,L^P) = v^* = (1-k)f_2(T^*,L^d) = (1-k)w^d$$

(L^P and L^d still depend only on B-T* and T*, respectively.)

Call the region of distribution from T = B/2 to T = T* Phase I. This is the region of no hiring. The size of the region depends on k, the supervision rate.

Clearly, for k = 0, T* = B/2. Phase I exists only at equality. Hiring begins the moment distribution becomes unequal.

Equally clearly, for k = 1, T* = B. Hiring can begin only where the landlord owns all the land, so the peasant's marginal product of land and the wage v* both equal zero. So for k = 1, Phase I applies to all distributions.

Assume k < 1, so T* < B. Then as distribution becomes more unequal, a point may come where the inequality begins to hold in (1.7.6). The landlord ceases to work directly on his own land, but merely supervises his employee. At that point, say the landlord owns T** acres, T* < T** $\leq$ B; and the peasant owns B-T**. The wage, v**, must be:

$$(3.3) \qquad w^P = f_2(B-T^{**},S^P) = v^{**} = (1-k)f_2(T^{**},H) = (1-k)w^d$$

S^P is the peasant's labor on his own land, while H is the peasant's labor on the landlord's land. So $L^P = S^P + H$. As will be seen, v** may be greater or less than v*, depending on k.

Call the region of distribution from $T = T^*$ to $T = T^{**}$ Phase II. As distribution becomes more unequal in Phase II, the landlord increasingly replaces his own applied labor with the peasant's labor, until at the end of Phase II, he only supervises.

Call the region of distribution from $T = T^{**}$ to $T = B$ Phase III. In Phase III, the landlord only supervises. The model of Sec. 1.8 instead of that of Sec. 1.7 now applies. In Phase III, from (1.8.2):

$$(3.4) \qquad w^p = f_2(B-T,S^p) = v \leq (1-k)f_2(T,H) \leq (1-k)w^d$$

with the two inequalities holding except for $T = T^{**}$, $v = v^{**}$.

Phase II must exist if $k < 1$. However, Phase III may not exist in a two-person model, -- depending on k, as well as the utility and production functions. That is, Phase II may extend all the way to complete inequality, where $T = B$. But for $k < 1$, Phase III will always exist for sufficient inequality in a large enough population.

7.4 Phase I. The Landlord Does Not Hire the Peasant[C]

In Phase I, high supervision costs keep the landlord from hiring the peasant. If supervision costs are high enough -- k = 1 -- then the landlord never hires the peasant, even when he owns all the land.

In Phase I, the landlord and peasant behave as two independent landowners like those in Sec. 1.5. So, as distribution becomes more unequal, the combined landlord-peasant economy behaves like the sum or average of the separate behavior of the landlord and peasant.

Assume for simplicity that k = 1, so Phase I applies to the whole range of distribution. Also assume there is enough land in the economy that the slope of the landlord's labor supply curve and the marginal product of the landlord's land approach zero when the landlord owns most of the land.

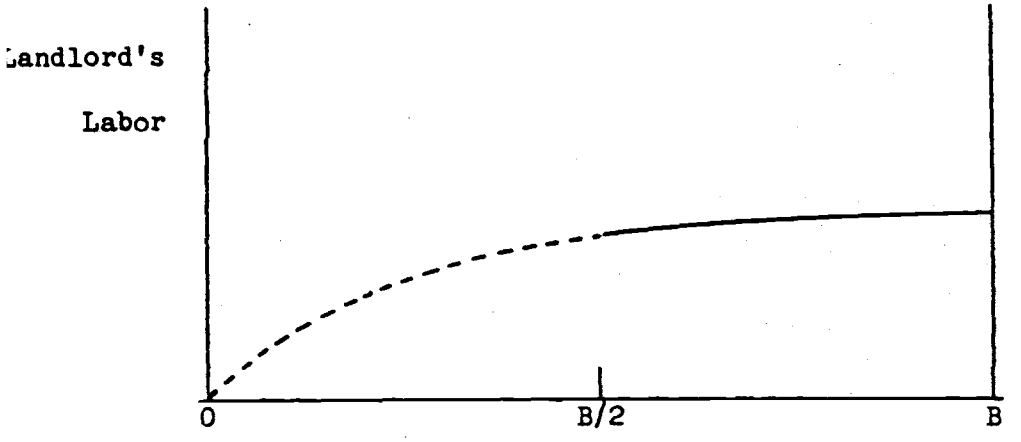
The landlord and the peasant easily combine graphically, with the landlord plotted from left to right, and the peasant from right to left, and the sum or average below.

Separate and combined results appear below for nine economic measures: labor, labor per acre, wage and marginal product of labor, output, rent and marginal product of land, average product of labor, average product of land, labor share of output, and consumption of food. Column 1 of Table 7.3 in section 7.6 summarizes these results.

(1) Labor

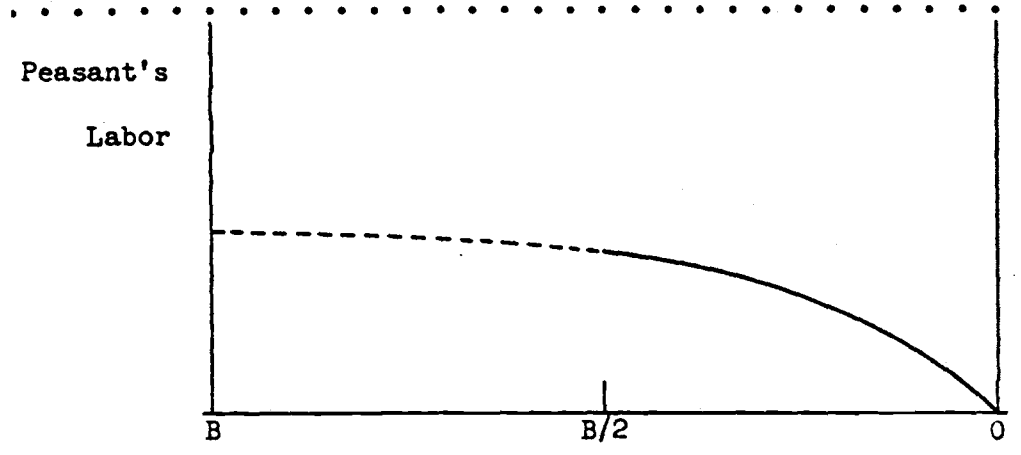
Figure 7.1 shows the landlord's and peasant's labor supplies, and their sum. The landlord's labor supply comes directly from Fig. 1.5, while the peasant's labor supply is the same "flipped over" to run from right to left.

As distribution becomes more unequal, the landlord's labor supply



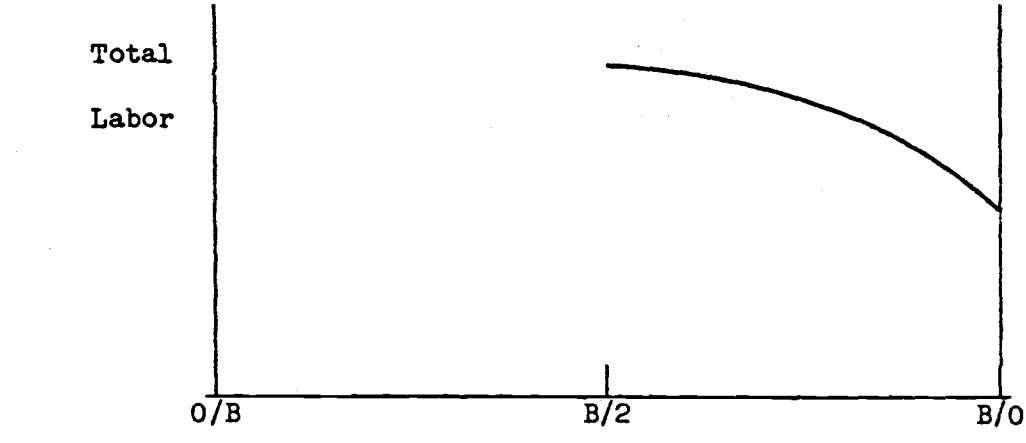
Landlord's Land Size -- T

Fig. 7.1a: Landlord's labor as function of land size.



Peasant's Land Size -- (B-T)

Fig. 7.1b: Peasant's labor as function of land size.



Distribution of Land: Landlord/Peasant

Fig. 7.1c: Total labor as a function of distribution.

increases, the peasant's labor supply falls, and the combined labor supply falls.

(2) Ratio of Labor to Land

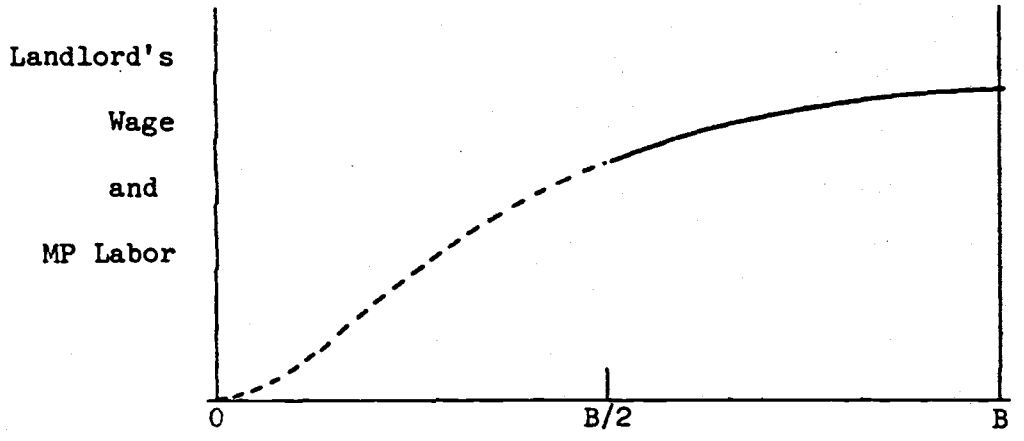
As distribution becomes more unequal, the ratio of labor to land falls on the landlord's land and rises on the peasant's land. The overall ratio of labor to land obviously falls, since total labor supply falls on a constant quantity of land, B.

However, the average ratio of labor to land may rise. For, as in Fig. 1.6, as land size increases, ratio of labor to land falls in a backwards "S". Assuming there is enough land in the economy so that the slope of the labor supply approaches zero as land approaches B, most of the "S" is "tail", convex to the origin. Consequently, for most of the distribution, the peasant's ratio of labor to land rises faster than the landlord's ratio falls, so the average ratio rises. In a model with a large population, mostly peasants, the average ratio of labor to land will certainly rise.

(3) Wage and Marginal Product of Labor

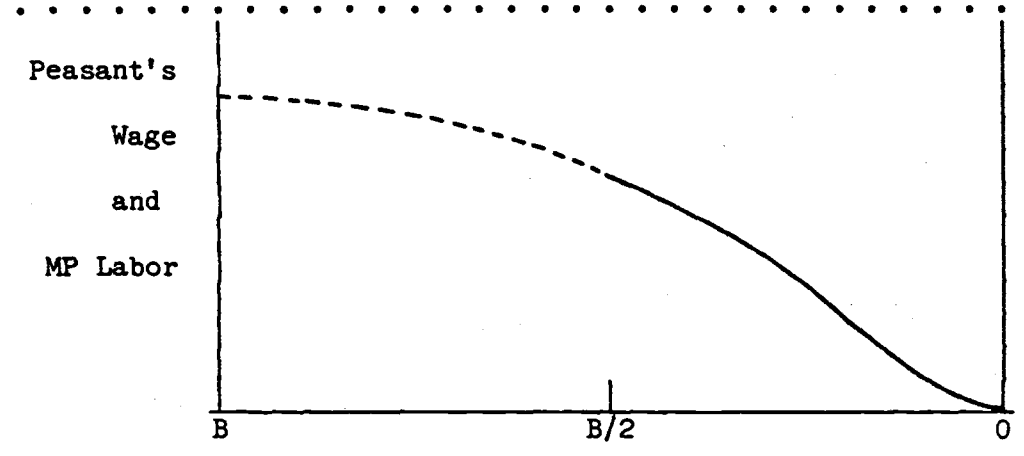
As distribution becomes more unequal, the landlord's wage and marginal product of labor rise and the peasant's wage and marginal product of labor fall. Fig. 7.2 shows the landlord's wage, from Fig. 1.7. It rises in an "S". By the assumption that slope of the labor supply approaches zero as land approaches B, most of the "S" is in the "head", so most of the curve is concave. The peasant's wage, in Fig. 7.2, follows the identical curve, "flipped over".

Fig. 7.2 also shows the average wage, which equals 1/2 the sum of the landlord's and peasant's wage. For most of the distribution, the peasant's wage falls faster than the landlord's wage rises, so the



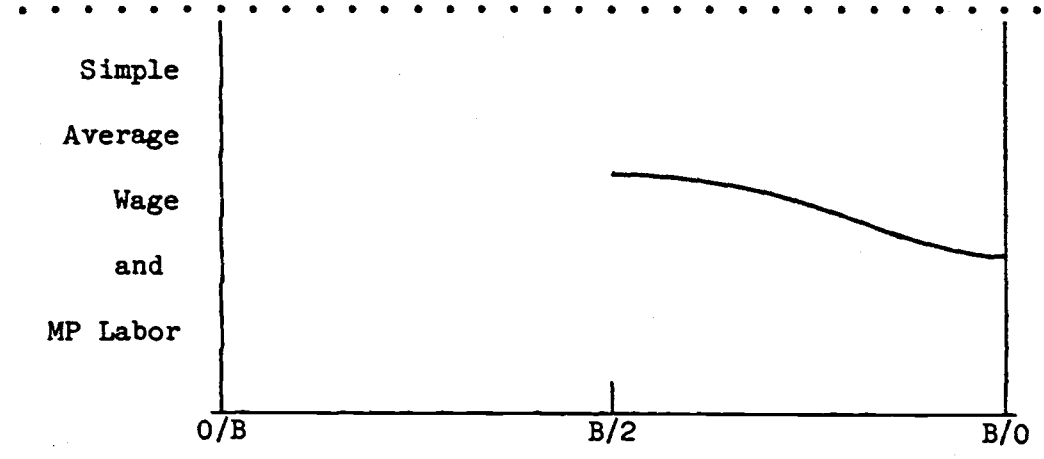
Landlord's Land Size -- T

Fig. 7.2a: Landlord's wage and MP labor as function of land size.



Peasant's Land Size -- (B-T)

Fig. 7.2b: Peasant's wage and MP labor as function of land size.



Distribution of Land: Landlord/Peasant

Fig. 7.2c: Simple average wage and MP labor as function of distribution.

average wage falls. In a model with a large population, mostly peasants, the average wage will invariably fall.

But how can average wage -- and marginal product of labor -- fall when labor supply falls on a constant quantity of land? In fact, while the simple average may fall, the weighted average rises, as it should. The simple average is:

$$SA = \frac{w^P + w^d}{2} = \frac{f_2{}^P + f_2{}^d}{2}$$

The weighted average is:

$$WA = \frac{w^P L^P + w^d L^d}{L^P + L^d} = \frac{f_2{}^P L^P + f_2{}^d L^d}{L^P + L^d}$$

The weighted average obviously rises. For it ranges from the wage and marginal product at equal distribution, up to the (higher) landlord's wage and marginal product at complete inequality.

(4) Output

As distribution becomes more unequal, the landlord's output rises while the peasant's output falls. Since the peasant's output falls faster than the landlord's output rises, combined output must fall. Landlord's, peasant's, and combined output appear in Figure 7.3. The landlord's output comes from Fig. 1.9, while the peasant's is the same, "flipped over".

(5) Marginal Product of Land and Rent

As distribution becomes more unequal, the landlord's marginal product of land and rent fall in a backwards "S", as shown in Fig. 1.8, and again in Fig. 7.4. By the assumption that there is enough land in the economy for the marginal product of land to approach zero, most of the "S" is

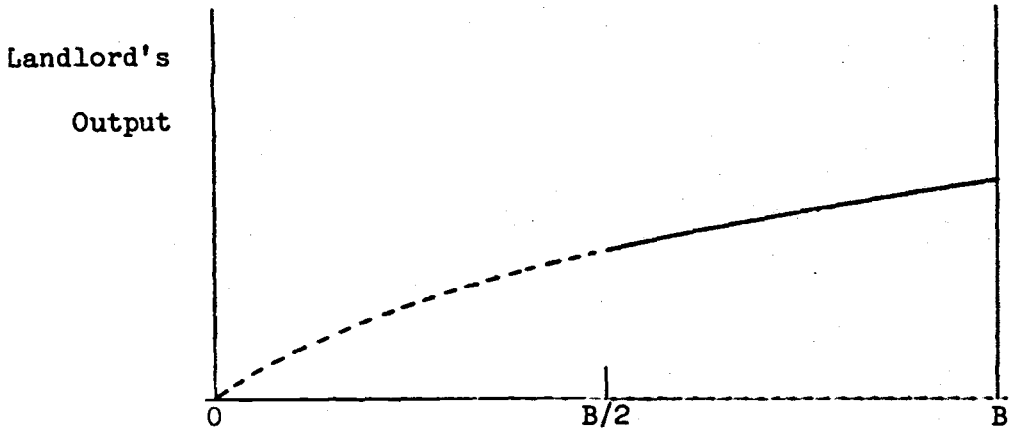


Fig. 7.3a: Landlord's output as function of land size.

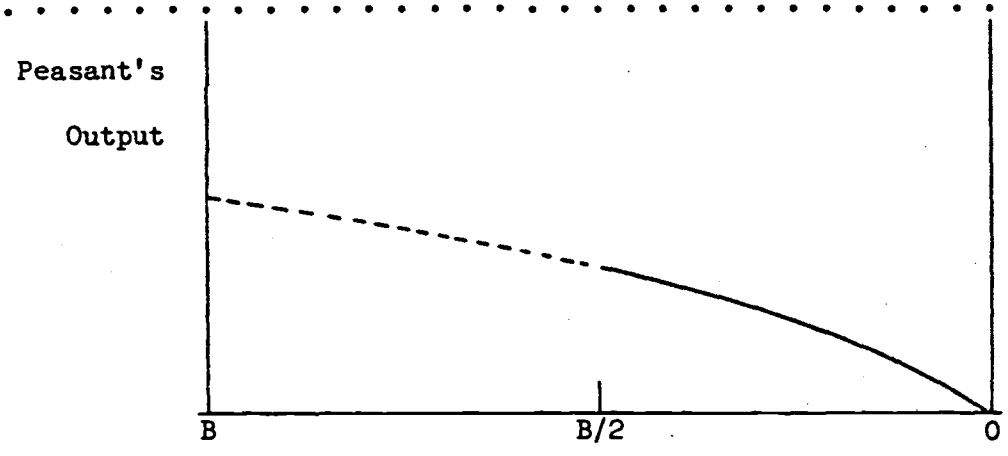


Fig. 7.3b: Peasant's output as function of land size.

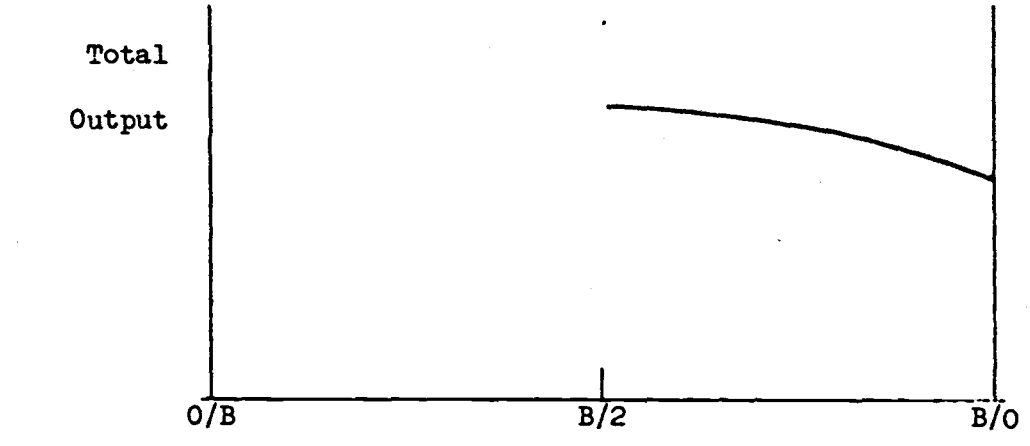


Fig. 7.3c: Total output as function of distribution.

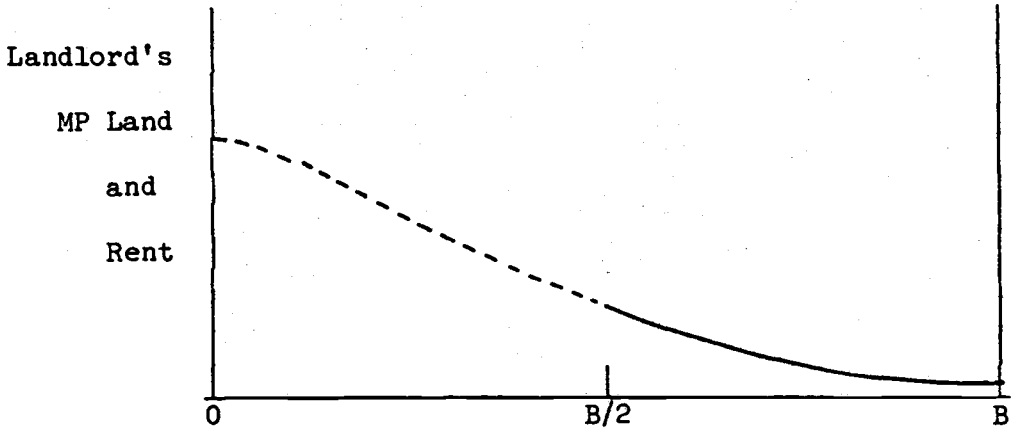


Fig. 7.4a: Landlord's MP land and rent as function of land size.

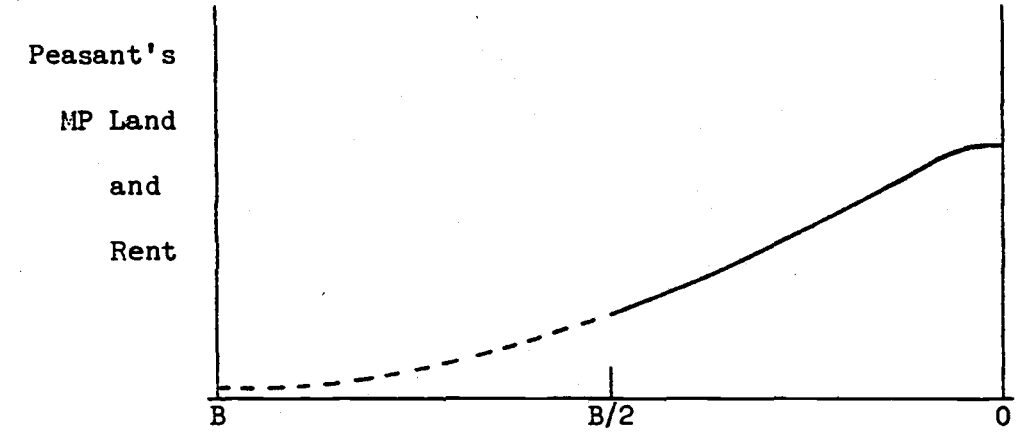


Fig. 7.4b: Peasant's MP land and rent as function of land size.

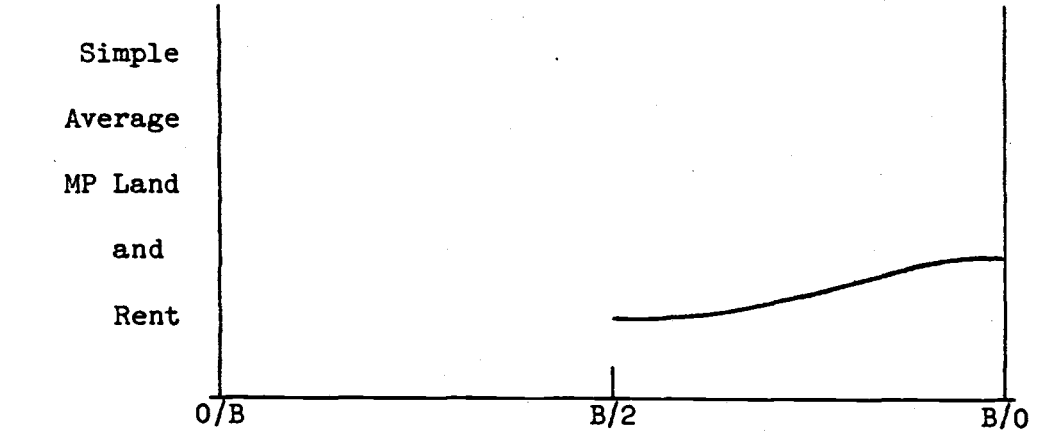


Fig. 7.4c: Simple average MP land and rent as function of distribution.

"tail", convex to the origin. The peasant's marginal product of land, in Fig. 7.4, follows the identical curve, "flipped over".

Fig. 7.4 also shows the average marginal product of land, which equals 1/2 the sum of the landlord's and peasant's marginal products of land. For most of the distribution, the average marginal product rises. In a model with a large population, mostly peasants, the average marginal product of land will invariably rise.

But although the simple average rises, the weighted average falls, as it should. The simple average is:

$$SA = \frac{f_1^P + f_1^d}{2} = \frac{r^P + r^d}{2}$$

The weighted average is:

$$WA = \frac{f_1^P(B-T) + f_1^d T}{B} = \frac{r^P(B-T) + r^d T}{B}$$

The weighted average ranges from the marginal product of land for B/2 acres, down to the lower marginal product of land for B acres.

(6) Average Product of Labor

As distribution becomes more unequal, the landlord's average product of labor rises, while the peasant's average product of labor falls. The average product of labor for the whole economy must rise, since it ranges from the average product for B/2 acres at equal distribution, to the higher average product for B acres when the landlord owns all the land.

(7) Average Product of Land (Output per Acre)

As distribution becomes more unequal, the landlord's average product of land falls while the peasant's rises. Average product of land for the

whole economy obviously falls, since output falls on a constant quantity of land.

The fact that the peasant's average product of land exceeds the landlord's average product means that the peasant's share of output exceeds his share of land. In other words, the distribution of production (and consumption) is not as unequal as the distribution of wealth. (Of course, by assumption, the landlord and peasant have equal labor skill, making the distribution of both inputs less unequal than the distribution of land alone.)

(8) Labor Share of Output

As distribution becomes more unequal, the landlord's labor share of output rises, while the peasant's falls. The labor share of output for the whole economy must rise since it ranges from labor share at equality to the higher landlord's labor share at complete inequality.

(9) Consumption of Goods (Food) = Ordinary Income

Consumption of course exactly coincides with production in Phase I, though not in other phases.

7.5 Phase II and Phase III with No Supervision Costs[C]

A supervision rate $k = 1$ represents one extreme: supervision costs so high that the landlord never hires the peasant, and the economy remains in Phase I for all distributions. At the opposite extreme lies a zero supervision rate, $k = 0$. In this case, the landlord hires the peasant the moment distribution diverges from equality, so Phase II begins at equality.

Assuming $k = 0$, what happens to the nine economic variables as distribution becomes more unequal? Graphics give the quickest and clearest answer.

First of all, what happens to labor and wage?

Since $k = 0$, there can be no supervisory labor. Therefore, all labor is "applied" in production on the peasant's or landlord's land. However, part (Phase II) or all (Phase III) the applied labor on the landlord's land is hired from the peasant.

Now consider how the labor applied to land increases as land size increases, given an unlimited quantity of labor available at a fixed wage, v. As shown in the models of Secs. 1.6, 1.7, and 1.8, the landowner will set the marginal product of labor equal to the wage: $f_2 = v$. So as land size increases, applied labor will increase, holding the marginal product of labor constant.

Consequently, for constant returns to scale in production, the ratio of applied labor to land stays constant as land increases. For increasing returns, the ratio rises, while for decreasing returns the ratio falls. From here on, to simplify the graphic arguments, assume constant returns. Fig. 7.5 shows applied labor demand as a function of land size, for four different given wages, $v_0 < v_1 < v_2 < v_3$.

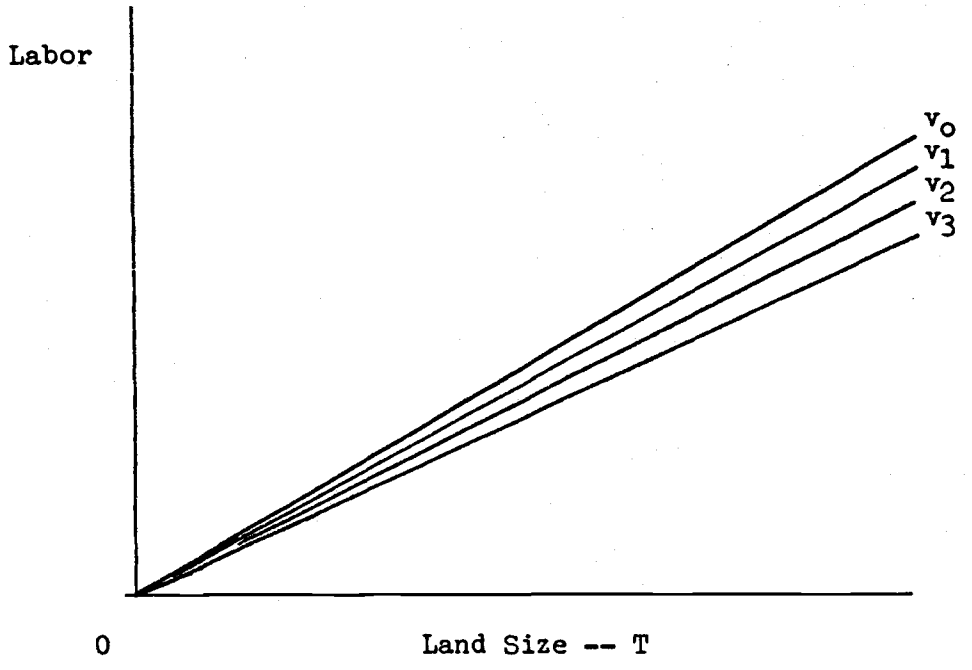


Fig. 7.5: Applied labor as a function of land size, at four different wages: $v_o < v_1 < v_2 < v_3$. Constant returns to scale in production.

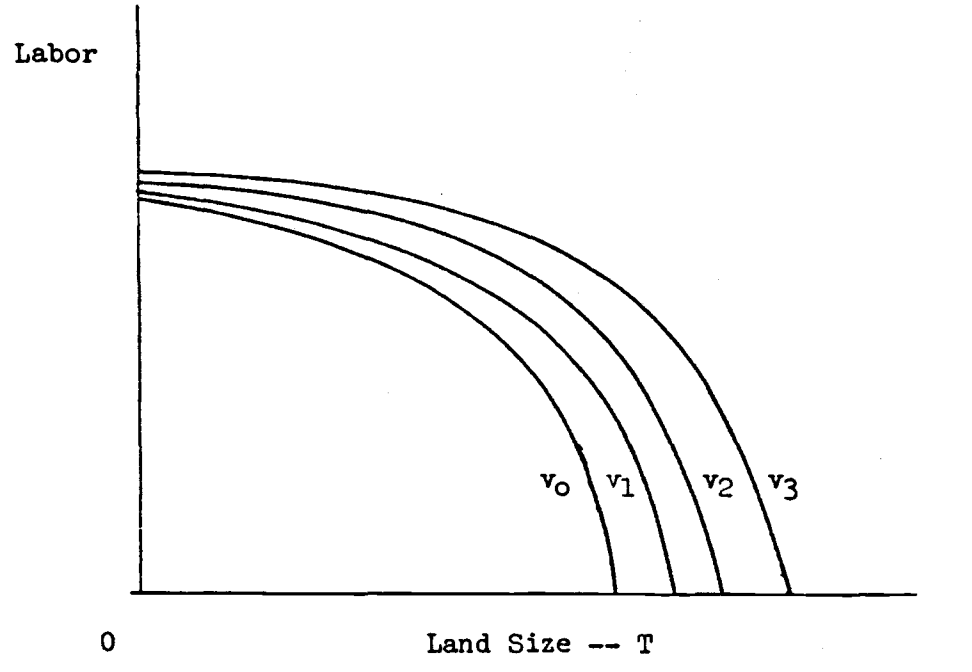


Fig. 7.6: Landowner's personal labor supply as a function of land size, at four different wages, $v_o < v_1 < v_2 < v_3$.

Applied labor demand increases in straight lines from the origin; the lower the wage, the steeper the slope. Assume a constant difference between wages.

An individual's personal labor supply depends on his income and his wage, as shown in Sec. 1.4. If his income increases, holding his wage constant, his personal labor supply will fall in a concave curve, as graphed in Fig. 1.2. So if a landowner's land increases, holding his wage constant, his income increases and his personal labor supply therefore falls, as shown in Sec. 1.6 and 1.7.

Fig. 7.6 shows a landowner's personal labor supply as a function of land size at the same four wages, $v_0 < v_1 < v_2 < v_3$. The higher the wage, the greater his personal labor supply at any given land size.

A landowner's personal labor supply minus his applied labor demand at a given wage v, gives hired labor supply or demand at wage v as a function of land size.

Fig. 7.7 shows a landowner's applied labor demand and his personal labor supply at a fixed wage, v_0, as a function of land size. The two curves cross at T_0. So at T_0, applied labor demand equals total labor supply. At T_0', the landowner's total labor supply goes to zero.

Fig. 7.8 shows the difference between the two curves. Hired labor supply falls in a concave curve to 0 at T_0. Hired labor demand rises in a convex curve from 0 at T_0. At T_0' there is a "kink" and hired labor demand becomes applied labor demand, a straight line.

Notice that at T_0, hired labor supply and hired labor demand must have the same slope, except for sign. Otherwise, hired labor supply is flatter than hired labor demand before the "kink", but steeper than hired labor demand beyond the "kink". This relation-

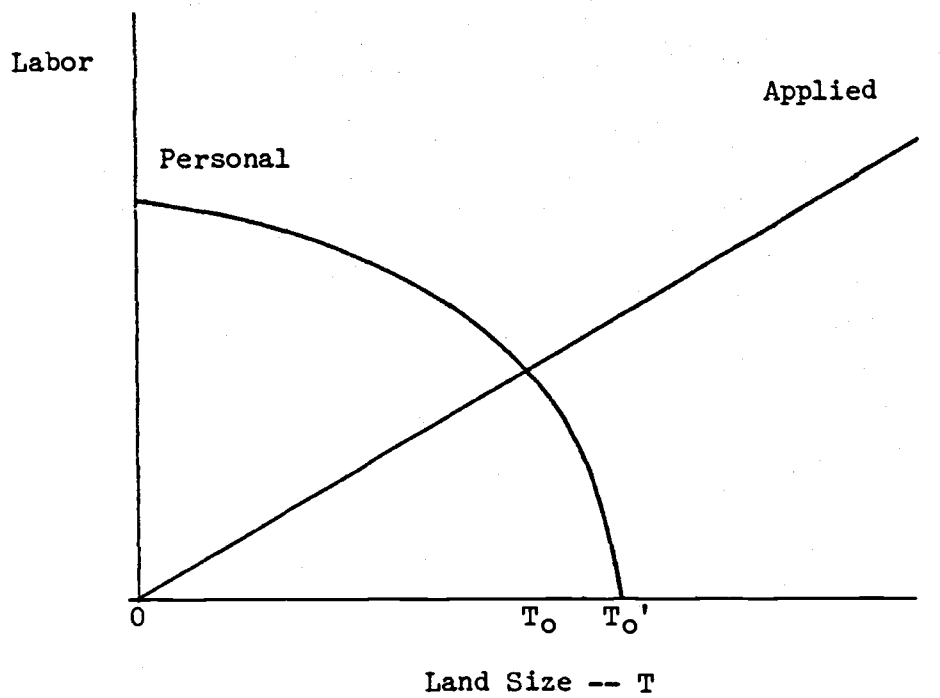


Fig. 7.7: Applied and personal labor supply at v_o.

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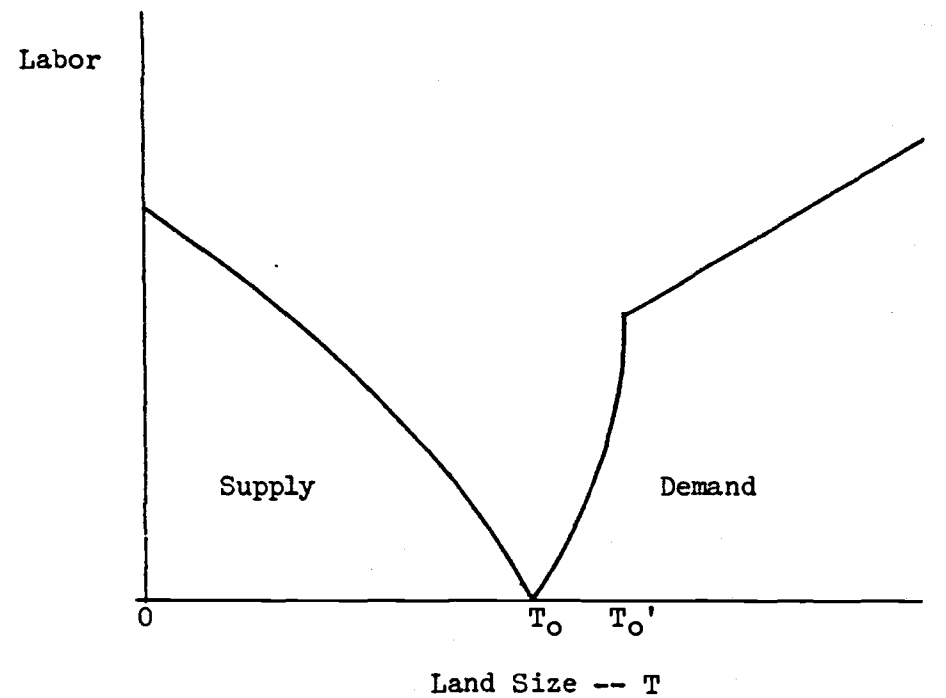

Fig. 7.8: Hired labor supply and demand at wage v_o.

ship becomes clearer if we "flip over" the hired labor supply at T_o, so it becomes an increasing supply as land goes from T_o to 0, as shown in Fig. 7.9. Supply must be less steep than demand before the "kink" because supply is convex while demand is concave. Supply must be steeper than demand beyond the "kink". For if it had the same slope, that would mean the landowner's personal labor supply remains constant instead of increasing as his land shrinks.

Fig. 7.10 shows a landowner's personal labor supply and applied labor demand at v_o and four higher and four lower wages, equally spaced. The applied labor and personal labor supply curves for v_o, v_1, v_2, and v_3 comes from Figs. 7.5 and 7.6 respectively.

Fig. 7.11 shows the corresponding hired labor supply and demand curves for these wages. Notice that the lower the wage, the steeper both hired labor supply and demand curves. Hired labor supply curves converge to the left as land size decreases. Hired labor demand curves diverge to the right as land size increases. So as land size increases, the spacing of supply and demand curves for hired labor increases. Greater spacing means greater sensitivity to a wage change; a given wage change will produce a greater hired labor change.

Landlord and Peasant Combined

Let v_o equal the marginal product of labor at B/2. So v_o equals the landlord and peasant's wage at equal distribution.

Then the landlord's demand curves for hired labor as a function of wages $v_o < v_1 < v_2 < v_3$ and land size from B/2 to B appear as in Fig. 7.12. The curves flatten as wage increases, and diverge as land increases, -- so the landlord's sensitivity to wage changes increases.

The peasant's supply curves of hired labor as a function of wage

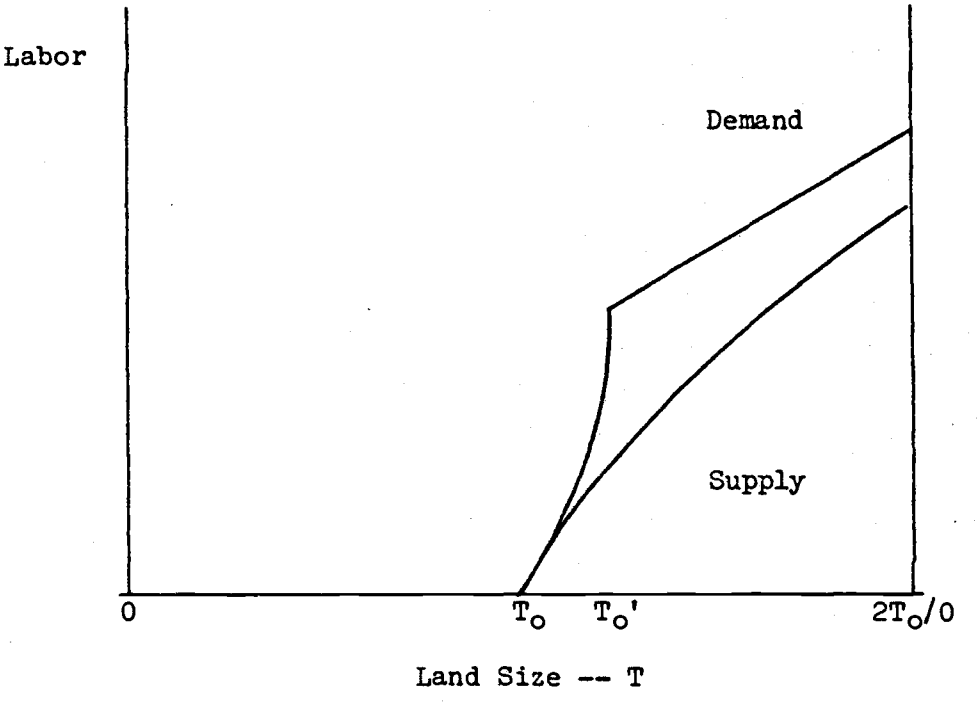

Fig. 7.9: Hired labor supply and demand at wage v_o. Supply rises as T falls from T_o to 0. Demand rises as T rises from T_o to $2T_o$. Note that demand always exceeds supply.

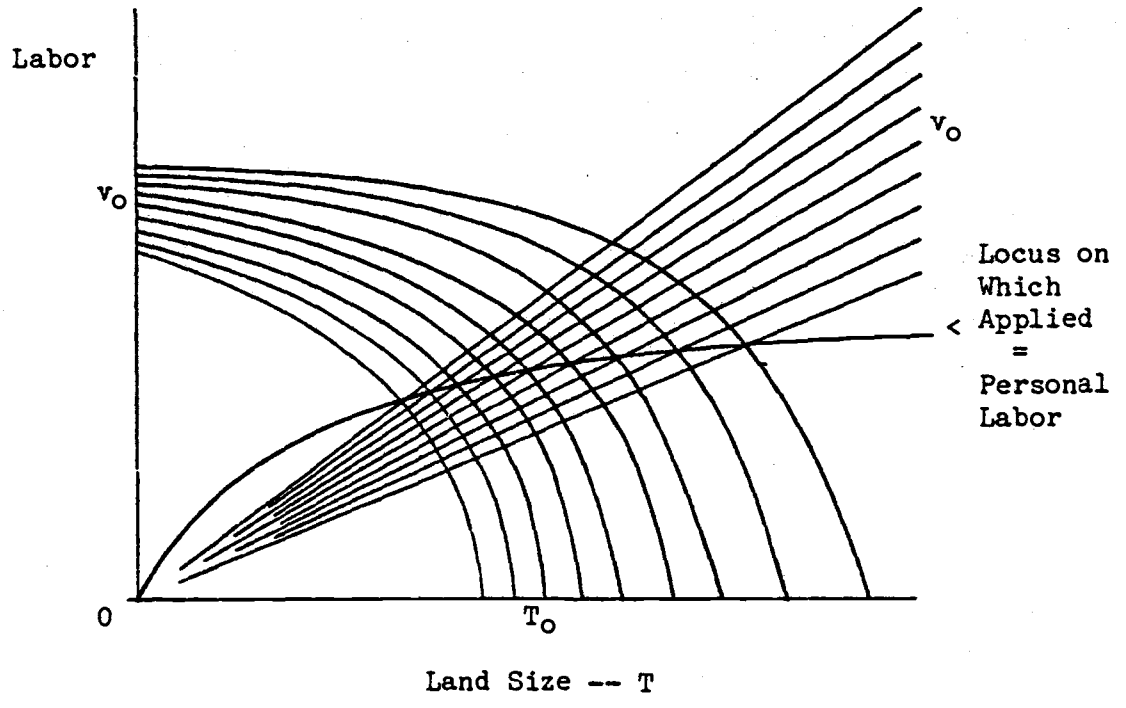

Fig. 7.10: Applied and personal labor as a function of land size for a number of wages both higher and lower than v_o.

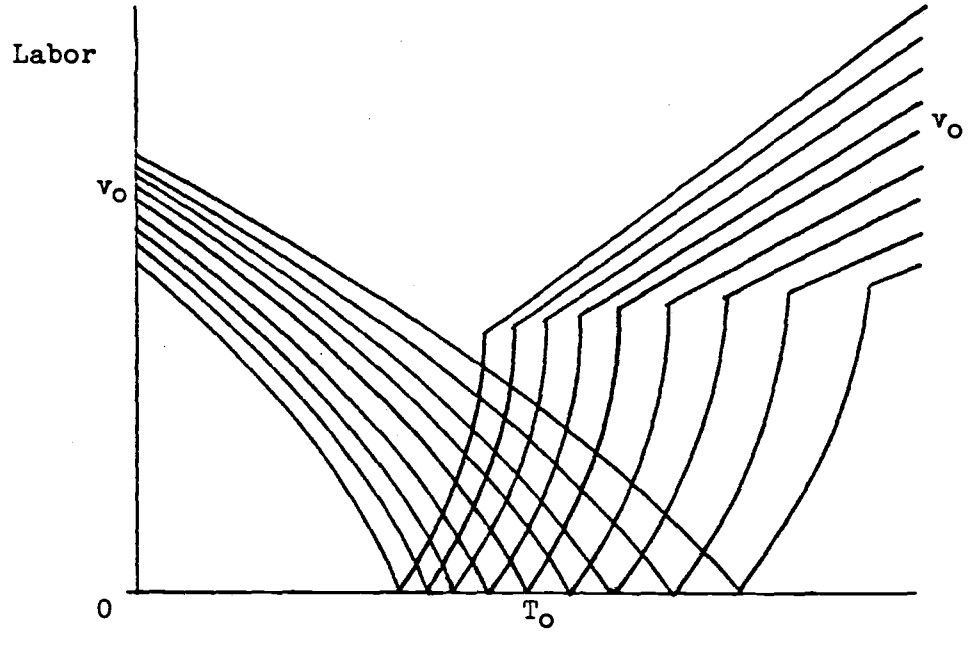

Fig. 7.11: Hired labor supply and demand as a function of land size for a number of wages both higher and lower than v_o.

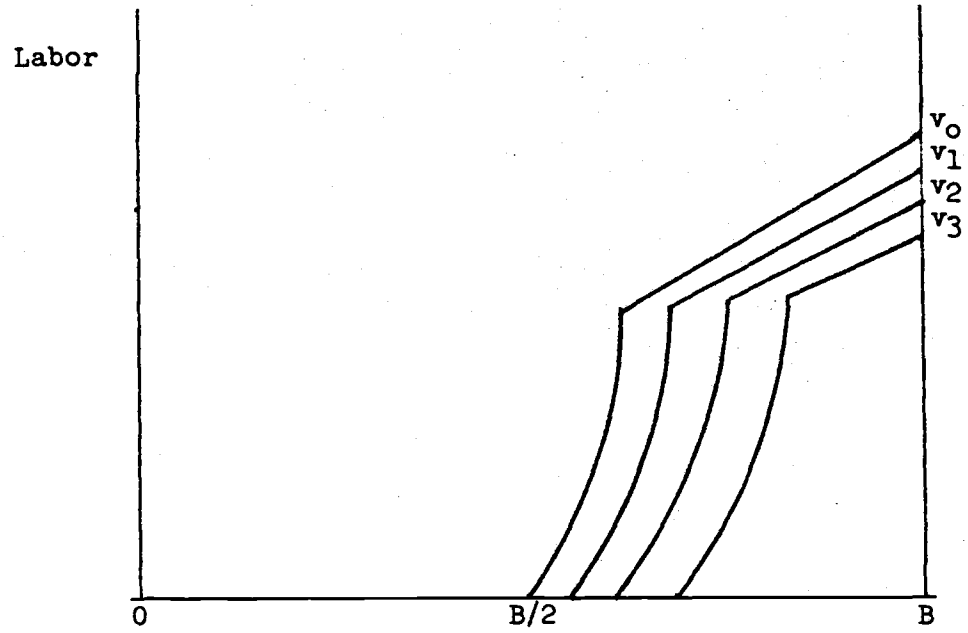

Fig. 7.12: Landlord's demand for hired labor as a function of land size at four wages: $v_o < v_1 < v_2 < v_3$. Note that the higher the wage, the lower the curve.

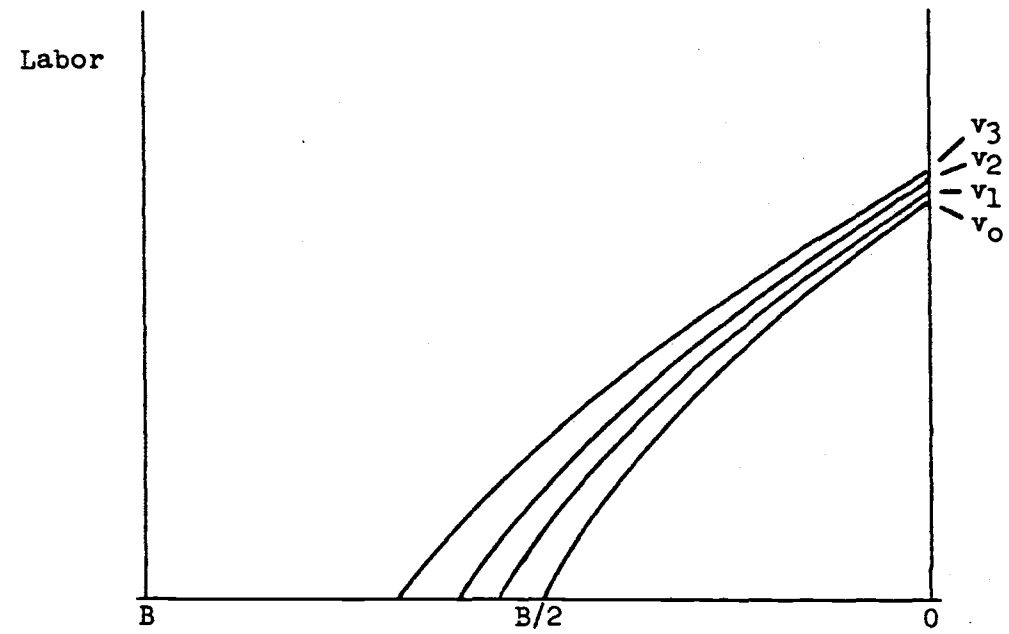

Fig. 7.13: Peasant's supply of hired labor as a function of land size (B-T), at four wages: $v_o < v_1 < v_2 < v_3$. Note that the higher the wage, the higher the curve.

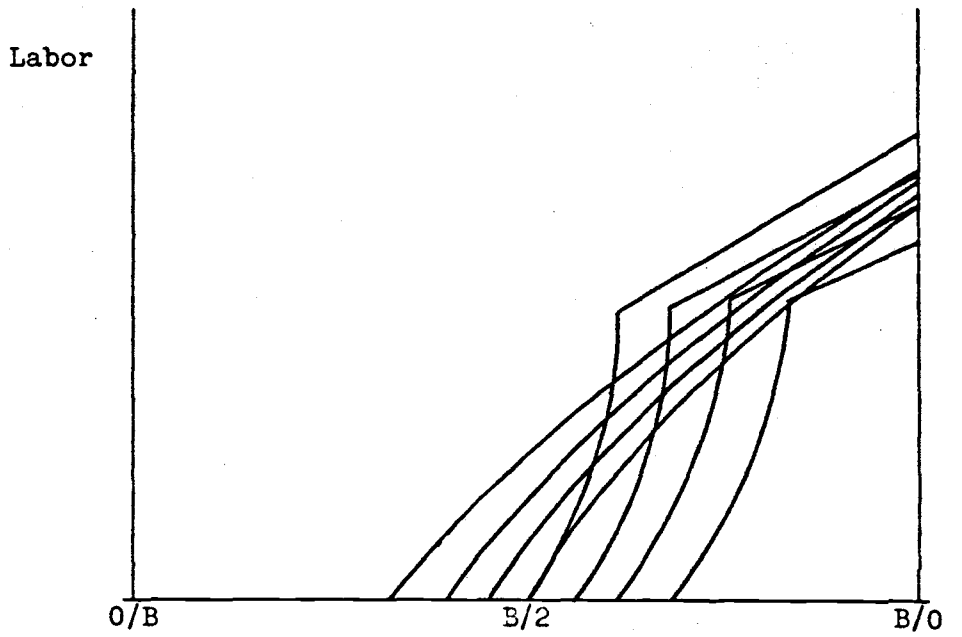

Fig. 7.14: Hired labor demand and supply curves combined.

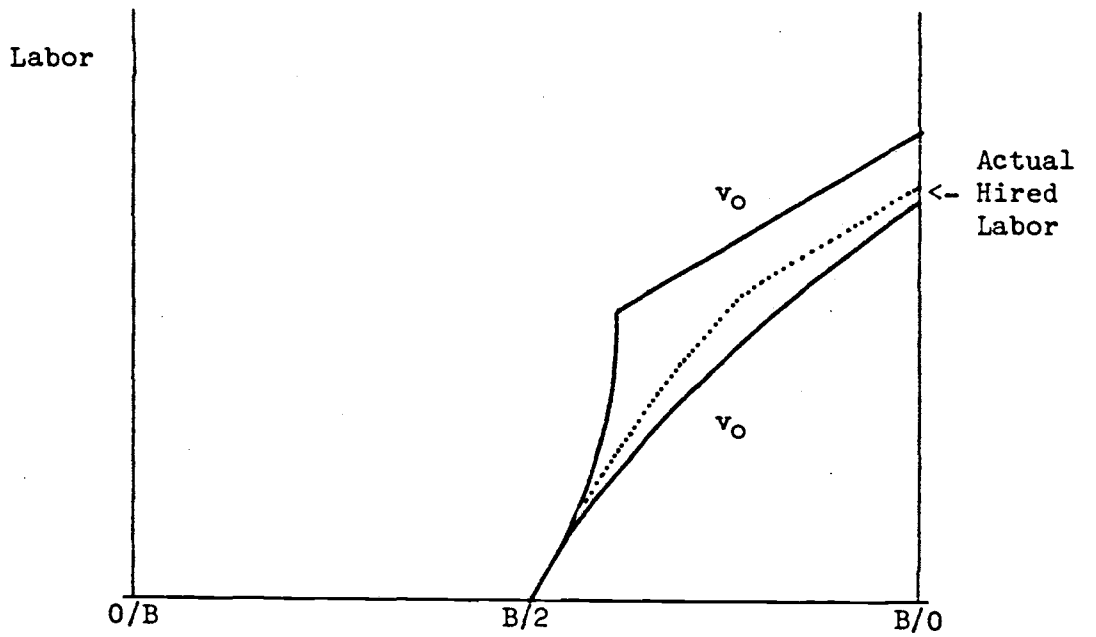

Fig. 7.15: Actual hired labor as a function of distribution, for k = 0. Actual hired labor (dotted line) falls between hired labor demand and supply at v_o, (solid lines), from Fig. 7.9.

and land size from B/2 to 0 appear as in Fig. 7.13. The curves also flatten as wage increases, but converge as land decreases, -- so the peasant's sensitivity to wage changes decreases.

Fig. 7.14 superposes the landlord's hired labor demand curves on the peasant's hired labor supply curves. Then the intersection of hired labor supply and demand curves for the same wage at each possible distribution from B/2 to B determines the actual quantity of hired labor.

Fig. 7.15 shows the actual quantity of hired labor (dotted line) which lies between the supply and demand curves for hired labor at v_o (solid lines). Note that the actual quantity hugs the supply curve, due to the landlord's much greater sensitivty to wage changes.

Phase II is the region where labor supply intersects the convex part of labor demand before the "kink". A glance at Fig. 7.14 shows the wage must rise here. For in Phase II, the landlord's demand for hired labor at any given wage rises faster than the peasant's supply. So wage must rise to equate supply and demand.

Beyond the "kink" lies Phase III. Here the wage falls, because the landlord's demand for hired labor at any given wage does not rise as fast as the peasant's supply. So wage must fall again to equate supply and demand.

However, the wage cannot fall all the way back to v_o, the wage at equal distribution. For hired labor at complete inequality must end up greater than supply at v_o but less than demand at v_o, as shown in Fig. 7.15.

So, for no supervision costs, increased inequality raises the wage for the economy. Since the wage equals the marginal product of land on both landlord and peasant's land, an increased wage means

a fall in the ratio of labor to land on both. Consequently, total labor supply, output, average and marginal product of land all fall. Average product of labor rises, as does labor share of output.

Table 7.2 summarizes the results for all variables for Phase II, Phase III, and both combined.

Table 7.2

Effect of Increased Inequality with No Supervision Cost

	PhaseII	PhasIII	Combind
1. Labor:			
Peasant's total personal: $L^P = S^P + H$	+	+	+
Self = applied: $S^P = A^P$	-	-	-
Hired: H	+	+	+
Landlord's total personal: $L^d = S^d$	-	0	-
Self: S^d	-	0	-
Applied: $S^d + H$ (II); H (III)	+	+	+
Total: $L^P + L^d = A^P + A^d$	-	+	-
2. Applied labor per acre:			
Peasant's = landlord's: $A^P/(B-T) = A^d/T$	-	+	-
3. Wage and MP labor (MPL):			
Peasant; wage = MPL: $w^P = f_2{}^P = f_2{}^d$	+	-	+
Landlord; wage: w^d	+	+?	+
MPL: $f_2{}^d = w^d$ (II)	+	-	+
4. Output:			
Peasant: F^P	-	-	-
Landlord: F^d	+	+	+
Total: $F^P + F^d$	-	+	-
5. MP land = rent:			
Peasant's = landlord's: $f_1{}^P = f_1{}^d$	-	+	-
6. Average product of labor:			
Peasant's = landlord's: $F^P/A^P = F^d/A^d$	+	-	+
7. AP land = output/acre:			
Peasant's = landlord's: $F^P/(B-T) = F^d/T$	-	+	-
8. Labor share of output:			
Peasant's = landlord's: $f_2{}^P A^P/F^P = f_2{}^d A^d/F^d$	+	-	+
9. Consumption of goods = ordinary income:			
Peasant's: $F^P + w^P H$	-	-	-
Landlord's: $F^d - w^P H$	+	+	+
Total: $F^P + F^d$	-	+	-

7.6 Phase II and III with Supervision Costs[C]

Suppose now the landlord must supervise the peasant at rate k, $0 < k < 1$.

Then the landlord will not start to hire the peasant at equal distribution, where each owns B/2, at wage v_o. Rather, from (3.2), the landlord will begin to hire the peasant where they own T* and B-T* respectively, $T^* > B/2$. Here, the peasant receives a wage of v*: $v^* < v_o$. But the landlord perceives a wage, including supervision costs, of $v^*/(1-k)$: $v^*/(1-k) > v_o$.

Throughout Phase II, the actual quantity of labor hired equals the quantity the peasant supplies at a wage, v, but which costs the landlord $v/(1-k)$, including supervision.

Fig. 7.16 shows the peasant's supply curves for hired labor. The curve for v_o is the same as in Fig. 7.13. There are additional curves for $v_{-4} < v_{-3} < v_{-2} < v_{-1} < v_o$, all derived by "flipping over" the hired labor supply curves of Fig. 7.11. The lower the wage, the lower and closer the curves. Assume $v_{-3} = v^*$, the wage at which the landlord starts to hire the peasant. The hired labor supply curve for $v_{-3} = v^*$ starts from the point B-T*.

Fig. 7.17 shows the landlord's demand curves for hired labor, at $k = 0$. The curves for v_o, v_1, v_2, and v_3 are the same as in Fig. 7.12, and the curve for v_4 is added from Fig. 7.11.

The landlord's demand curve for labor at v_2 starts from T*. So $v_2 = v^*/(1-k) = v_{-3}/(1-k)$. In addition, assume that $v_o = v_{-4}/(1-k)$, and $v_4 = v_{-2}/(1-k)$. (As a little manipulation shows, this means assuming $k = 1/2$.)

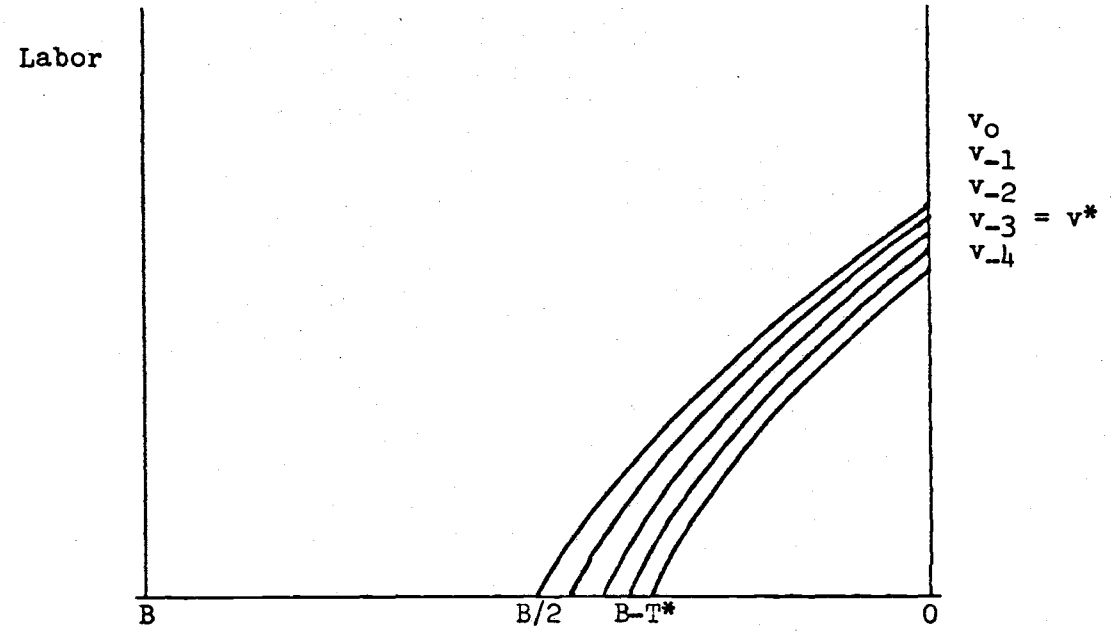

Fig. 7.16: Peasant's supply curves for hired labor, at wages $v_o > v_{-1} > v_{-2} > v_{-3} > v_{-4}$. $v_{-3} = v^*$, the wage at which the landlord starts to hire the peasant.

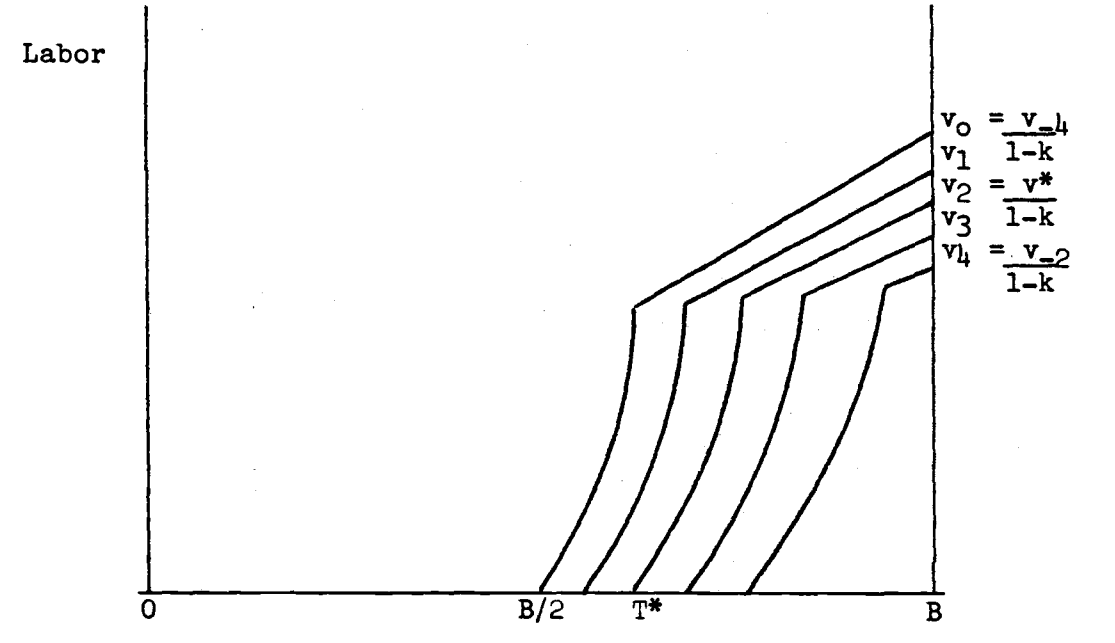

Fig. 7.17: Landlord's demand curves for hired labor at $v_o < v_1 < v_2 < v_3 < v_4$ for $k = 0$. $v_o = v_{-4}/(1-k)$. $v_2 = v^*/(1-k) = v_{-3}/(1-k)$. $v_4 = v_{-2}/(1-k)$.

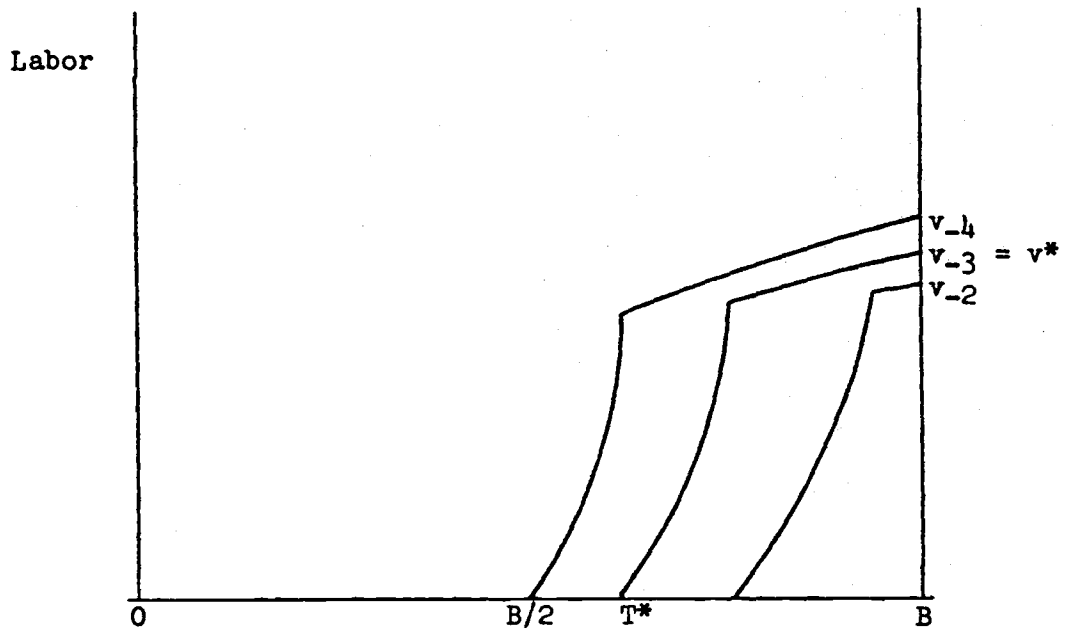

Fig. 7.18: Landlord's demand for hired labor, for $k > 0$, at wages $v_{-4} < v_{-3} < v_{-2}$. $v_{-3} = v^*$.

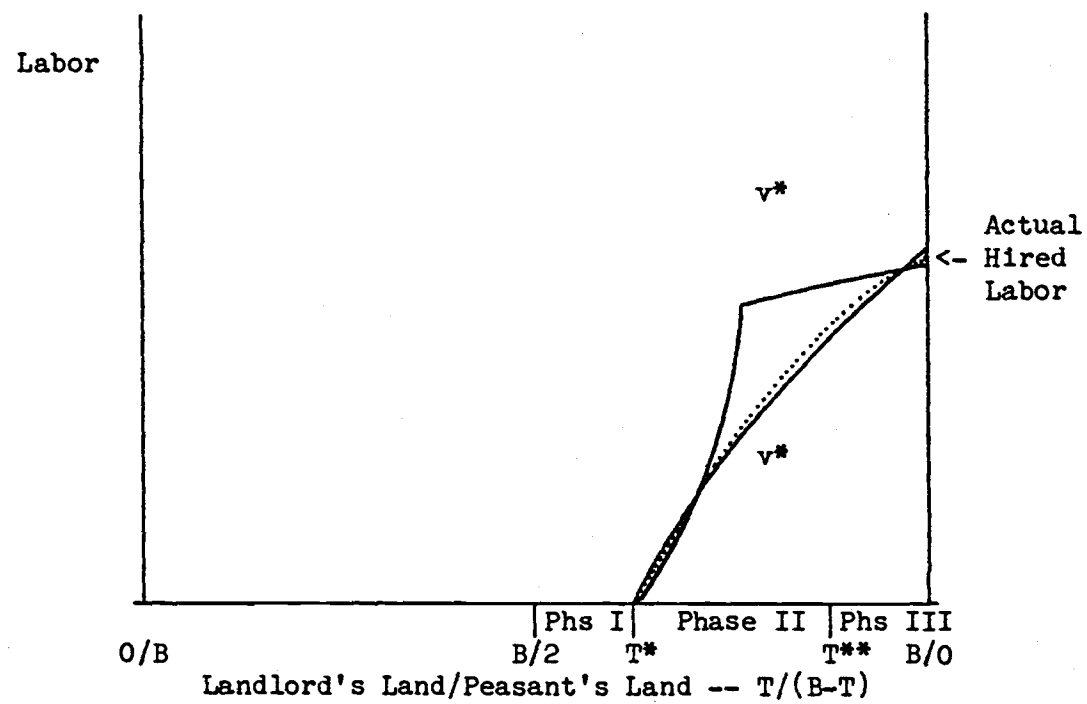

Fig. 7.19: Actual hired labor as a function of distribution, for $k > 0$. Actual hired labor (dotted line) falls between hired labor demand and supply at $v^* = v_{-3}$, (solid lines). Compare Fig. 7.15.

Since for $k > 0$, in Phase II, the landlord perceives a wage (including supervision) of $v/(1-k)$, then the landlord's demand curves for labor at v with supervision must coincide with his curves for labor at $v/(1-k)$ without supervision. So in Phase II, the hired labor demand curves for v_0, v_2, and v_4 at $k = 0$ become the hired labor demand curves for v_{-4}, $v_{-3} = v^*$, and v_{-2} for $k > 0$, as shown in Fig. 7.18. These curves now lie much further apart than the landlord's demand curves when $k = 0$.

In Phase III, for $k = 0$, the landlord's hired labor demand curves radiate in straight lines from the origin, as shown in Fig. 7.12; the higher the wage, the flatter the slope. But for $k > 0$, the landlord must supply his own labor in proportion to hired labor, since $L^d = kH$. Since his own labor converges asymptotically toward a maximum, D, his hired labor demand curves at different wages must eventually converge toward D/k. Therefore, these curves must flatten and converge towards each other as well.

Fig. 7.18 shows the landlord's hired labor demand curves for $k > 0$ at three wages, v_{-4}, $v_{-3} = v^*$, and v_{-2}. The curves are convex and divergent up to the "kink"; then they are concave and convergent.

Fig. 7.19 shows the supply and demand curves at v^*, (solid lines). It also shows actual hired labor as a function of distribution (dotted line). Actual hired labor traces the intersection of the peasant's supply curves of Fig. 7.17 and the landlord's demand curves of Fig. 7.18.

The supply and demand curves for v^* cross twice.

First they loop across each other to form a narrow "needle's eye" at the beginning of Phase II. This happens because the higher the wage, the flatter both supply and demand curves. So, at zero hired labor, the

landlord's demand curve for hired labor at v* -- which matches the demand curve for $v_2 = v^*/(1-k) > v^*$ -- is flatter than the peasant's supply curve of hired labor at v*. But since demand curves are convex while supply curves are concave, the curves meet again and cross at the top of the "needle's eye".

The curves cross again to form a triangle beyond the "kink", toward the end of Phase III, because here the slope of the peasant's hired labor supply curve greatly exceeds the flattening slope of the landlord's demand curve.

The actual hired labor curve always lies between supply and demand at v*. Where supply exceeds demand, -- in the "needle's eye" and again in the "triangle" at the end of Phase III, -- the wage must fall below v*. Where demand exceeds supply, -- between the "needle's eye" and the "triangle", -- the wage must rise above v*. So, as distribution becomes more unequal, the wage first falls a bit below v*, rises above it, and falls below it again. The final wage comes out much lower than the final wage in the absence of supervision requirements: about v_{-4}, as opposed to v_2.

Notice that, even more than in the case of no supervision requirement, actual hired labor sticks close to the peasant's supply curve, -- due to the landlord's greater sensitivity to wage changes.

As shown in Section 7.3, the larger the supervision requirement, k, the lower the wage, v*, and the larger the landlord's land, T*, when hiring starts. The graphic presentation shows that, as k increases from zero, first the "needle's eye" appears and grows. Then the "triangle" appears and grows. Finally, the "needle's eye" and "triangle" merge, so that the wage never rises back to v*. For large enough k, the

wage falls continuously from v*. Fig. 7.20 shows the supply and demand curves for successively lower v*, starting with v_o, corresponding to successively higher k, starting with k = 0.

What happens to total labor supply?

In Phase II, total applied labor obviously falls when the wage rises, as it will for k = 0. But what about the beginning of the "needle's eye", where the wage falls a bit before rising again for small k, or falls continually for large k? For a fall in the wage means the ratio of labor to land rises on both peasant's and landlord's land.

Despite the fall in the wage, total applied labor still falls, because the landlord's ratio of applied labor to land lies below the peasant's -- so greater inequality increases the proportion of land with the lower ratio of applied labor to land. Fig. 7.21 shows how, at a constant wage, greater inequality lowers total applied labor. A consequent fall in the wage will reduce the amount total applied labor falls. As shown above, the greater k, the greater the fall in the wage, -- but also the greater the difference in landlord's and peasant's applied labor ratios. So total applied labor in Phase II always falls.

In Phase III, total applied labor equals the peasant's total personal labor supply. For k = 0 or small k, the peasant's personal labor supply increases, since the effect of his reduced wealth outweighs the effect of the small wage decrease. For larger k, the more rapid decline in the wage with greater inequality drives down the peasant's personal labor supply. So total applied labor in Phase III rises for small k, and falls for large k.

In combination, total applied labor must always fall. For, as shown in the previous section, even for k = 0, applied labor does not rise

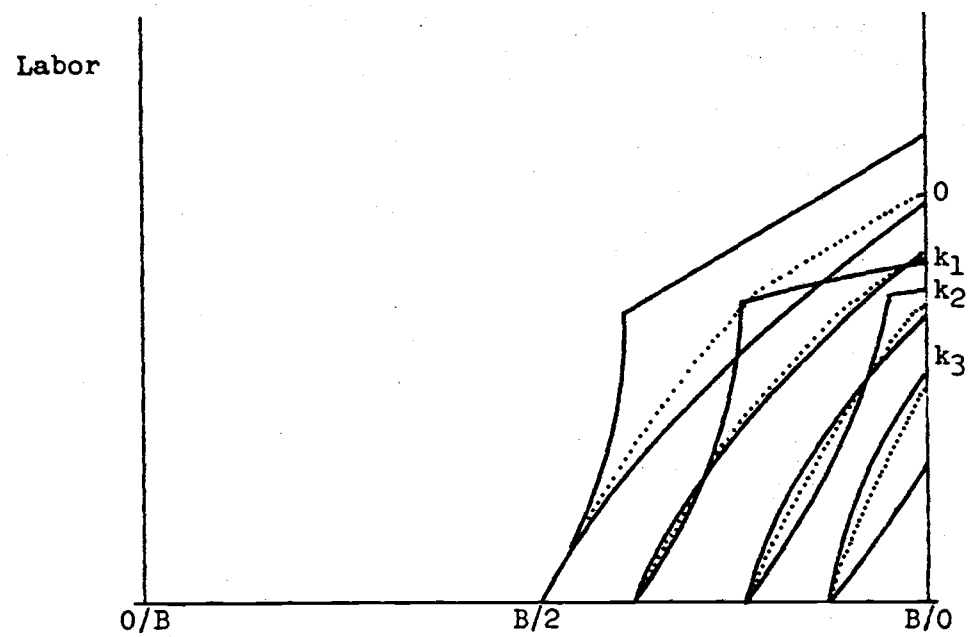

Fig. 7.20: Actual hired labor (dotted lines) as a function of distribution for four values of k: $0 < k_1 < k_2 < k_3$. Hired labor supply and demand curves for the same values (solid lines).

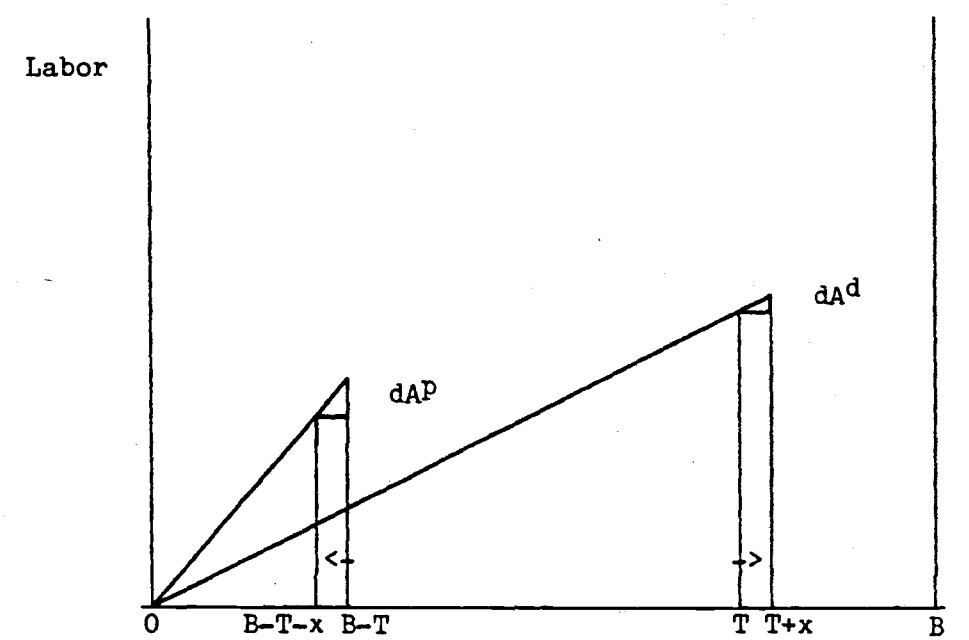

Fig. 7.21: Effect of transferring x acres of land from peasant to landlord, holding wage constant, when landlord has a lower ratio of applied labor to land. $dA^P + dA^d < 0$.

back to its value at T = B/2. The same must hold with supervision costs, even though for small k, total applied labor increases in Phase III.

Total labor, including supervision, will behave pretty much like total applied labor -- particularly in a more general model where peasants greatly outnumber landlords.

Table 7.3 summarizes the effects of increased inequality, given a cost of supervision. The first column shows the effects in Phase I, which extends from equality to the point the landlord owns T*. The second column shows Phase II, the third, Phase III, and the fourth, the combined results. Phases II, III, and combined also show the difference in effect when the supervision rate is small, and when it is large.

Greater inequality with a small supervision rate has mostly the same effects as with none. Greater inequality with a large rate has almost the same effects as in Phase I, with a prohibitive rate. Only in Phase I, hired and supervisory labor do not exist, and the landlord's applied labor increases.

The effect of greater inequality on wage and labor supply usually obviously determines the effect on the many other variables of Table 7.4. When in doubt the effect can be guessed by considering the limiting cases where k = 0, or k = 1. It is assumed, as seems plausible, that for a high supervision rate, the simple averages favor the peasant, while the weighted averages favor the landlord. Sometimes help comes from the tables of partial derivatives with respect to land size and wage in Sections 1.5 through 1.8.

Table 7.3

Effect of Increased Inequality with Supervision Cost

	Phase I	PhaseII		PhasIII		Combind	
		lo*	hi*	lo	hi	lo	hi
1. Labor:							
Peasant's total personal: $L^P = S^P + H$	-	+	-	+	-	+	-
Self = applied: $S^P = A^P$	-	-		-		-	
Hired: H	+	+		+		+	
Landlord's total personal: $L^d = S^d + kH$	+	-		+		-	+
Self: S^d	+	-		0		-	
Supervisory: kH	0	+		+		+	
Applied: $A^d = S^d + H$ (II); H (III)	+	+		+		+	
Total; personal, appled: L^P+L^d, A^P+A^d	-	-		+	-	-	
2. Applied labor per acre:							
Peasant's: $A^P/(B-T)$	+	+-†	+	+	+	-	+
Landlord's: A^d/T	-	+-	+	+	-	-	
Simple average: $[A^P/(B-T) + A^d/T]/2$	+	+-	+		+	-	+
Overall (wghted avrage): $[A^P + A^d]/B$	-	-		+	-	-	
3. Wage and MP labor (MPL):							
Peasant: $w^P = f_2^P = f_2^d/(1-k)$ (II)	-	-+†	-	-		+	-
Landlord; wage: $w^d = [f_2^d - w^P]/k$	+	-+	-	+		+	
MPL: $f_2^d = w^d$ (II)	+	-+	-	-	+	+	
Diffrnc; wage: $w^d-w^P = kw^P/(1-k)$ (II)	+	-+	-	+		+	
MPL: $f_2^d - f_2^P = kw^d$	+	-+	-	+		+	
Smp avg; wage: $(w^d+w^P)/2$	-	-+	-	-		+	-
MPL: $(f_2^d+f_2^P)/2 = w^P + kw^d/2$	-	-+	-	-		+	-
Wgt avg: $\frac{w^PL^P + w^dL^d}{L^P + L^d}$, $\frac{f_2^PA^P + f_2^dA^d}{A^P + A^d}$	+	+		-	+	+	
4. Output:							
Peasant: F^P	-	-		-		-	
Landlord: F^d	+	+		+		+	
Total: $F^P + F^d$	-	-		+	-	-	
5. MP land = rent:							
Peasant: f_1^P	+	+-	+	+		-	+
Landlord: f_1^d	-	+-	+	+	-	-	
Simple avg: $(f_1^P + f_1^d)/2$	+	+-	+	+		-	+
Weighted avg: $[f_1^P/(B-T) + f_1^d/T]/B$	-	-		+	-	-	

* "lo": "low supervision rate, k". "hi": "high supervision rate, k".

† "-+": "- then +"; just "+", k = 0. "+-": "+ then -"; just "-", k = 0.

Table 7.3, cont'd

	Phase I	PhaseII		PhasIII		Combind	
		lo	hi	lo	hi	lo	hi
6. Average product of applied labor:							
Peasant: F^p/A^p	-	-+	-	-		+	-
Landlord: F^d/A^d	+	-+	-	-	+	+	
Overall: $(F^p + F^d)/(A^p + A^d)$	+	+		-	+	+	
7. AP land = output/acre:							
Peasant: $F^p/(B-T)$	+	+-	+	+		-	+
Landlord: F^d/T	-	+-	+	+	-	-	
Overall: $(F^p + F^d)/B$	-	-		+	-	-	
8. Labor share of output:							
Peasant: $f_2^p A^p/F^p$	-	-+	-	-		+	-
Landlord: $f_2^d A^d/F^d$	+	-+	-	-	+		
Overall: $(f_2^p A^p + f_2^d A^d)/(F^p+F^d)$	+	+		-	+	+	
9. Consumption = ordinary income:							
Peasant's: $F^p + w^p H$ F^p (I)	-	-		-		-	
Landlord's: $F^d - w^p H$ F^d (I)	+	+		+		+	
Total: $F^p + F^d$	-	-		+	-	-	

7.7 Effect of Increased Supervision Costs[C]

An increase in the required supervision rate, k, first of all moves the boundaries of Phases II and III to greater inequality, -- as apparent in Fig. 7.20. It also appears from Fig. 7.20 that the greater k, the smaller the range of Phase II.

Within Phase I, an increase in the required supervision rate, k, has no effect whatsoever.

In both Phases II and III, an increase in k lowers the peasant's wage. From this it follows that hired labor and the peasant's total labor both fall, while the peasant's applied labor rises. An increase in k also raises the effective cost of labor to the landlord, so that the landlord's applied labor falls.

In Phase III, total applied labor equals the peasant's total labor supply, so total applied labor obviously falls. In Phase II, total applied labor must fall for substantial inequality and/or a high rate of supervision, -- since then the increase on the peasant's land cannot possibly make up for the decrease on the landlord's land. It seems plausible by continuity that total applied labor also falls for small k and slight inequality.

Sometimes the effect of increasing k depends on the existing level of k and degree of inequality. For example, when k is very small, an increase will raise supervisory labor; for large k, the opposite will happen. (Supervisory labor does not exist for $k = 0$, or for k high enough to preclude hiring, in Phase I.)

Other effects can be derived from the effects on wage and labor supply, with the help of the partial derivatives with respect to k and wage in Tables 1.5, 1.6, and 1.7. It is assumed, as plausible, that simple averages

favor the peasant. Weighted averages necessarily favor the landlord.

Table 7.4 shows how increased k affects the nine selected economic variables.

Table 7.4

Effect of Increased Supervision Cost, Given Inequality

	PhaseII		PhasIII		Combind	
	lo*	hi	lo	hi	lo	hi
1. Labor:						
Peasant's total personal: $L^P = S^P + H$	-		-		-	
Self = applied: $S^P = A^P$	+		+		+	
Hired: H	-		-		-	
Landlord's total personal: $L^d = S^d + kH$	+		+	-?	+	+?
Self: S^d	+?	+	0		+	
Supervisory: kH	+	?	+	-?	+	-?
Applied: $A^d = S^d + H$ (II); H (III)	-		-		-	
Total; personal, applied: $L^P + L^d$, $A^P + A^d$	-		-		-	
2. Applied labor per acre:						
Peasant's: $A^P/(B-T)$	+		+		+	
Landlord's: A^d/T	-		-		-	
Simple average: $[A^P/(B-T) + A^d/T]/2$	+		+		+	
Overall (weighted average): $[A^P + A^d]/B$	-		-		-	
3. Wage and MP labor (MPL):						
Peasant: $w^P = f_2^P = f_2^d/(1-k)$ (II)	-		-		-	
Landlord; wage: $w^d = [f_2^d - w^P]/k$	+		+	?	+	?
MPL: $f_2^d = w^d$ (II)	+		+		+	
Difference; wage: $w^d - w^P = kw^P/(1-k)$ (II)	+		+	?	+	?
MPL: $f_2^d - f_2^P = kw^d$	+		+		+	
Simple avg; wage: $(w^d+w^P)/2=(2-k)w^P/(1-k)$ (II)	-		-		-	
MPL: $(f_2^d+f_2^P)/2 = w^P + kw^d/2$	-		-		-	
Weigtd avg: $\frac{w^PL^P + w^dL^d}{L^P + L^d}$, $\frac{f_2^PA^P + f_2^dA^d}{A^P + A^d}$	+		+		+	
4. Output:						
Peasant: F^P	+		+		+	
Landlord: F^d	-		-		-	
Total: $F^P + F^d$	-		-		-	
5. MP land = rent:						
Peasant: f_1^P	+		+		+	
Landlord: f_1^d	-		-		-	
Simple avg: $(f_1^P + f_1^d)/2$	+		+		+	
Weighted avg: $[f_1^P/(B-T) + f_1^d/T]/B$	-		-		-	

* "lo": "low supervision rate, k". "hi": "high supervision rate, k".

Table 7.4, cont'd

	PhaseII		PhasIII		Combind	
	lo	hi	lo	hi	lo	hi
6. Average product of applied labor:						
Peasant: F^p/A^p	-		-		-	
Landlord: F^d/A^d	+		+		+	
Overall: $(F^p + F^d)/(A^p + A^d)$	+		+		+	
7. AP land = output/acre:						
Peasant: $F^p/(B-T)$	+		+		+	
Landlord: F^d/T	-		-		-	
Overall: $(F^p + F^d)/B$	-		-		-	
8. Labor share of output:						
Peasant: $f_2^p A^p/F^p$	-		-		-	
Landlord: $f_2^d A^d/F^d$	+		+		+	
Overall: $(f_2^p A^p + f_2^d A^d)/(F^p+F^d)$	+		+		+	
9. Consumption of goods = ordinary income:						
Peasant's: $F^p + w^p H$	-		-		-	
Landlord's: $F^d - w^p H$	+?		-		?	
Total: $F^p + F^d$	-		-		-	