CHAPTER 6

WEALTH, FIRM SIZE, AND RISK

The models so far have not treated risk explicitly, though risk-averse behavior presumably contributes to transactions costs. But the models do in fact lead to some significant and novel predictions about differences in behavior toward risk between richer and poorer people, larger and smaller firms.

Sec. 6.1 summarizes the predictions of the models. Sec. 6.2 draws implications for the long-standing controversy over whether richer people and larger firm managers are--or should be--less or more risk-averse.

6.1 Predictions

Suppose we define a good called "security", which depends on two aspects of risk: a) the proportional variability or "riskiness" of net income, as measured by standard deviation over expected value, and b) the proportional skewness of risk, as measured by the third moment (positive for an upward skew, and negative for a downward skew) over expected value--"proportional third moment". Security varies inversely with riskiness. It varies directly with proportional third moment. So people may seek riskiness with an upward skew: a small chance of large gain balancing a large chance of small loss. They may avoid riskiness with no skew or a downward skew: a small chance of large loss balancing a large chance of small gain.

Notice that this definition of security explains the gambler who buys fire insurance not, a la Friedman and Savage, by the relative <u>size</u> of fire risk and odds at the track, but by the relative <u>skew</u> of the risks. Fire risk is skewed down, and track odds are skewed up. Odds in risky endeavors like inventing and wild-catting are presumably skewed upwards, possibly making them very attractive to people with relatively little to lose.

Sec. 6.4 shows formally the properties of riskiness and proportional third moment. There's a close relationship between them: most actions that lower riskiness--like buying insurance, pooling risks, or reducing leverage--also bring proportional third moment closer to zero. So when risk is skewed upward, there's a loose trade-off between lower riskiness and higher positive proportional third moment.

The models of the preceding chapters suggest that richer people and managers of larger firms consume more security, as the empirical evidence seems to show. They are also less likely to be in a position of high positive proportional third moment, as again the empirical evidence seems to show.

In brief, assuming security is a normal good, consumption will increase with wealth and firm size unless the cost of producing security rises drastically. Richer people and bigger firms have many advantages and some disadvantages in production of security: They hire better people (but supervise them less), they are less leveraged, and they enjoy economies of scale in risk-pooling. But the latter two advantages, plus sheer size, reduce upward skew of riskiness and hence the attractiveness of entrepreneurial risk-taking or innovation. Sec. 6.3 reviews these arguments.

Of course security is a <u>future good</u>, although riskiness and proportional third moment can only be measured after the fact. So people actually consume <u>perceived discounted</u> security. As shown in Sec. 6.5, the fall in discount rate with wealth or firm size may greatly affect perceived security (or insecurity).

In a couple of plausible situations, as shown in Sec. 6.5, the lower the discount rate, the higher the perceived riskiness:

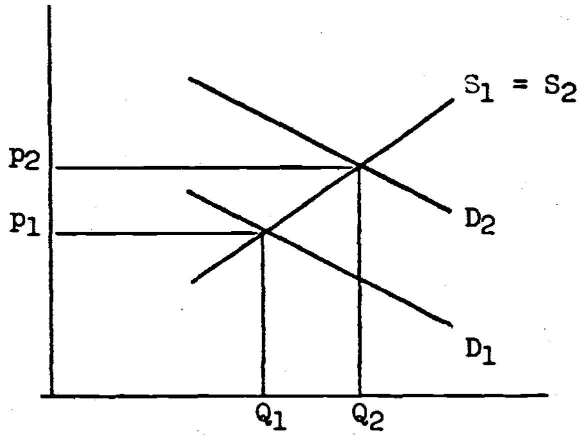
a) receipt of income preceeds liability. This is par excellence the case when someone borrows money to make an investment due to pay off before the loan must be repaid. Such a loan may seem far less risky to a desperate or rosy-glassesed small businessman than to his fish-eyed banker. For the businessman gives proportionally more weight to the near receipt than to the more remote liability.

b) an investment yields a stream of income (or other benefits) that grows increasingly risky with distance in the future--an apt description of virtually all investments. If riskiness rises fast enough with distance into the future, such an investment may look so much riskier to a richer than a poorer person, that the poorer person can outbid a richer one. That is, a richer person adds so much larger a risk premium to his discount rate as to value the investment lower than the poorer person. Examples of such investments might be used cars and machinery, nearly-depleted oil fields, etc.

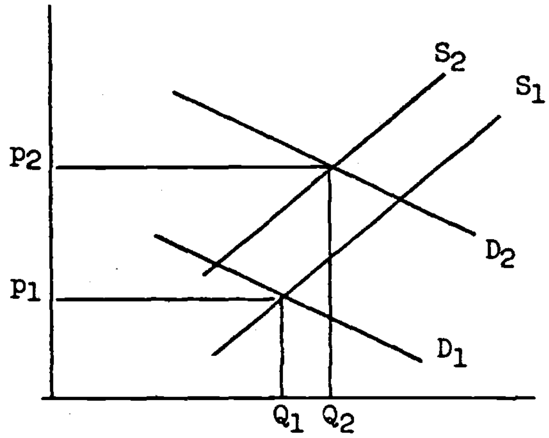
On the other hand, a lower discount rate raises instead of lowers perceived security when riskiness arises primarily from illiquidity. An illiquid asset is one whose market is "thin". Hence a seller may have to wait a long time to find a buyer offering a good price. A lower discount rate gives richer people greater waiting power, and hence a comparative advantage in owning illiquid assets: Old Masters, country estates, controlling blocks of stock, etc.

6.2 Wealth, Firm Size, and Risk-Aversion

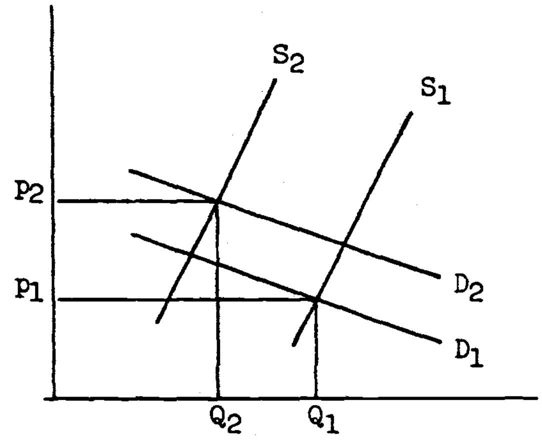
What of the controversy over whether richer persons and managers of



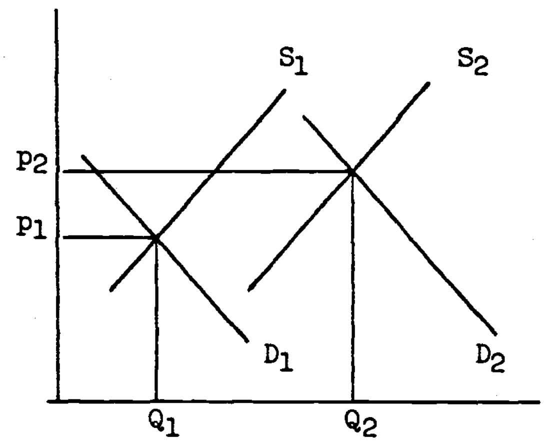
Case 1: Supply does not shift but demand shifts out.



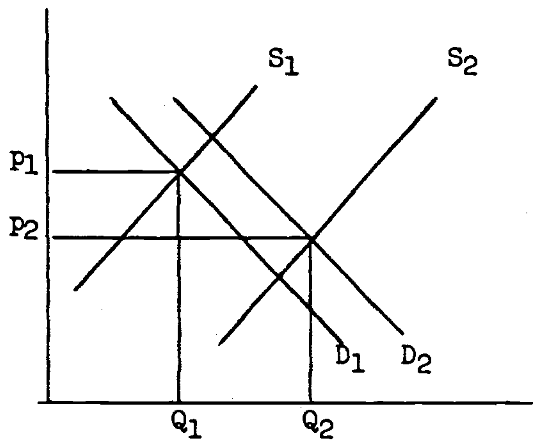
Case 2a: Supply shifts in slower than demand shifts out.



Case 2b: Supply shifts in faster than demand shifts out.



Case 3a: Supply shifts out slower than demand shifts out.



Case 3b: Supply shifts out faster than demand shifts out.

Fig. 6.1: Changes in price and quantity of non-market goods, assuming demand always shifts outwards with wealth or firm size, but supply may shift in or out or not at all.

larger firms are or should be less or more risk-averse?

Partly the controversy depends on definitions, and partly on analysis.

a. "Richer persons or larger firm managers are less risk-averse because they can undertake larger absolute risks." Perfectly true, but not very interesting. For they may still take smaller proportional risks. The analysis above assumed "security" to depend on proportional risk: "riskiness".

b. If risk-aversion depends simply on the demand for security, then the mere assumption that security is a normal good--one whose demand curve shifts outwards with wealth or firm size--makes richer persons and larger firm managers more risk-averse. If security is a superior good, that makes them very much more risk-averse.

c. Risk-aversion could depend on the implicit price of security. If the demand curve shifts out faster than the supply curve then the implicit price rises and resources move to increased production of security at the expense of other production. This is clearly what Caves and others mean when they argue that managers of larger firms are more risk-averse. On the other hand, if the supply curve shifts out faster than the demand curve, the implicit price falls, and resources move away from security to production of other goods. This could be called decreased risk-aversion. In either case, however, consumption of security increases.

d. It's possible to argue that, due to their superior risk-pooling ability, richer persons and managers of larger firms can better undertake investments that are riskier in isolation. This argument does not truly concern risk-aversion at all, but the technology of risk-pooling. It amounts to a claim that risk-pooling offers such huge economies of scale that an individually riskier investment adds less riskiness to a larger portfolio than an individually safer investment adds to a smaller portfolio. So richer persons and managers of larger firms can take on individually riskier investments, and yet still have safer portfolios.

Does risk-pooling truly offer such dramatic economies of scale? Four points cut against the argument.

First of all, economies of scale in risk-pooling require some statistical independence of investments. Independence may hold for routinized investments like insurance policies or small bank loans. But more unusual investments probably depend heavily on common factors like quality of management or the state of the stock market.

Second, to save on supervision costs, richer persons and larger firm managers prefer bigger individual investments. This preference limits the number of investments in the pool, and hence the gains from pooling.

Third, for the same reason, they also prefer investments that require less supervision--probably making them intrinsically less risky.

Finally, consider employees' incentives. Suppose that employees get punished for losses. But the more investments they take on, the greater the probability of <u>some</u> losses, --even though standard deviation falls. So taking on more investments makes employees' personal riskiness higher and proportional third moment more negative. They logically limit investments, at a sacrifice in gains from pooling.

These points also weaken any claim that larger firms should innovate more. Innovations are not highly poolable investments, and they probably require close supervision and good employee incentives.

e. Sometimes the argument in d. may go one step further: "Richer persons and managers of larger firms have so great an advantage in risk-pooling that they can take on such risky investments as to enjoy both lower riskiness and a higher return on investment."

This argument violates the assumption of Chp. 1, necessary for general equilibrium, that transactions costs must eventually outweigh any economies of scale. And the argument is internally contradictory anyway. For richer individuals and managers of larger firms have a comparative advantage in risk-pooling for exactly the same reason they get a lower return on investment: transactions costs. Transactions costs simultaneously keep them from investing their money at higher rates of return, and keep poorer persons and smaller firm managers from getting together to pool risks. To put it another way, risk-pooling and other activities showing economies of scale are just some of the many ways richer people and larger firm managers mitigate transactions costs.

6.3 Implications of Previous Chapters for Consumption of Security

Definition and Measurement of Security:

As described in Sec. 6.1, assume an individual's security depends on the riskiness of his expected income stream (wages plus profit or share of profit), and on the skewness, as measured by proportional third moment (PTM).

Note that the riskiness and PTM of an individual's income may differ considerably from those of the firm he owns or manages. As an extreme example, if an individual draws the entire expected net revenue of his firm as salary, expected profit is zero and riskiness of profit is infinite. Yet the riskiness of the owner's income might be quite low. And the riskiness and PTM of a firm manager's income includes the possibility he may be fired--a fact which should not directly affect the riskiness or PTM of the firm's profits. (It may affect riskiness indirectly by making the manager act more risk-averse.)

Obviously, the riskiness and PTM of firm owners' and managers' total income affects their decisions about the firm's operations, more than the riskiness and PTM of the firm's profit in the abstract. That poses a problem of measurement, since data on riskiness and PTM may exist only for firms. But assume, as seems reasonable, that riskiness and PTM for firms and for their owners and managers largely coincide.

Consumption of Security:

Demand for security obviously must increase with wealth and firm size. For security is surely a normal good. Assuming future-orientation increases with wealth, as argued in Chp. 4, security may even be a superior good.

So unless supply of security falls drastically with wealth or firm size, consumption must increase. (Given transactions costs, the supply of leisure does in fact fall drastically with wealth and firm size. Hence the assumption of Chp. 1 that, even though leisure is a normal good, consumption of leisure falls with wealth or size of firm managed.)

Why might supply of security rise or not fall more than demand rises as wealth or firm size increase?

The Supply of Security--Factors Reducing Supply:

A simple assumption underlies the models presented so far: Less well supervised employees produce less from given land. Likewise, the expected value of rent collected from tenants falls as the supervision rate falls. Yet there are only 24 hours in a day, and richer people value their time more highly. So richer landowners necessarily supervise less. Consequently, per acre output and rent fall with wealth.

Obviously, lower supervision of tenants--resulting in a higher default rate--increases the riskiness of rent, and lowers an already negative PTM.

Assume the same holds for employees. Lower supervision of employees increases the riskiness of output.

And lower supervision also logically lowers PTM. Less well supervised employees more often blunder or steal than achieve an unexpected feat of productivity (for which they would receive little or no reward). Innovation presumably has a positive PTM. But innovation also requires strong motivation and close attention, --liable to fall as supervision falls.

PTM logically falls with wealth or firm size for another reason: market size or physical environment increasingly limit possible proportional gains. For example, a struggling small businessman might have a slight chance of "making it big", possibly multiplying his wealth a hundredfold, versus a large chance of losing his small shirt--for a net positive PTM. And a small oil company has a much better chance to make a relatively big strike than does a big oil company. But the manager of a large corporation faces a good chance of a modest appreciation in the value of his shares of stock, versus a slight chance of losing his job--for a net negative PTM.

These factors all shift the supply curve of security inwards.

The Supply of Security--Factors Increasing the Supply:

As shown in Chapters 2 and 3, richer persons or managers of larger firms respond to their shortage of supervisorial time in many ways other than just reducing supervision. These responses simultaneously reduce riskiness and either raise PTM, and/or move it closer to zero.

(1) Better Employees:

First of all, richer persons hire more skilled and reliable employees, at a higher wage. In so doing, they conserve supervisorial time at a sacrifice in net instead of gross output. But hiring better employees surely reduces riskiness and raises PTM as well.

(2) Lower Leverage:

Second, richer persons generally choose lower leverage, both operating leverage and financial leverage--again as a means of conserving supervisorial time. But, as shown in Sec. 6.6, lower leverage brings lower riskiness. It also brings PTM closer to zero, raising negative PTM but lowering positive PTM, as shown in Sec. 6.4.

a. Operating leverage. By definition, operating leverage equals gross income divided by net income--gross minus operating costs.

Operating leverage falls with firm size due to rising comparative advantage in activities with low intrinsic labor-intensity, as shown in Chapter 3. So costs of hired labor and other current costs fall as a proportion of output. In general, richer persons and larger firm managers prefer more durable assets--assets showing a proportionally high ratio of income flow to depreciation costs.

b. Financial leverage. Financial leverage in Sec. 2.7 equals gross income divided by gross income minus rental payments. More generally, it is gross income divided by gross income minus debt service. (Debt service can be analyzed as rent plus installment purchase, so the difference isn't that great.)

Financial leverage falls with wealth because richer landowners can conserve their labor (direct or supervisory) by renting less additional land. (Corporations can similarly conserve managerial labor by taking on less debt.) In fact, as Sec. 2.7 shows, if rent per acre were fixed--as it would be in a world without transactions costs--richer landowners would actually rent so much less additional land as to operate smaller farms! Only if per acre rent falls as leverage falls (and possibly also as quantity of rented land increases) do acreage of rented land and farm size increase with wealth. This fall in rent reflects the assumption that it costs more per acre to supervise a small, highly-leveraged rental agreement than a large, less-leveraged one.

Of course a landowner faces a tradeoff between operating and financial leverage. He can lower his operating leverage by renting more land, --at the cost of raising his financial leverage.

(3) Economies of Scale:

Any activity probably offers some economies of scale in risk-pooling. Moreover, as shown in Chapter 3, richer individuals and larger firms enjoy a general comparative advantage in activities offering economies of scale, --presumably including those offering particularly great economies of scale in risk-pooling. And given minimum (ordinary) economies of scale in any activity, richer individuals and larger firms can diversify at lower cost.

As shown in Sec 6.4, risk-pooling reduces riskiness and moves PTM closer to zero, raising a negative PTM but lowering a positive one.

(4) Lower Supervision Cost Activities:

As shown in Chapter 3, richer individuals and larger firms enjoy a comparative advantage in activities with lower supervision costs. If, as seems plausible, riskier activities require more supervision, then richer individuals and larger firms have a comparative advantage in less risky activities. Less risky activities plausibly show a PTM closer to zero.

Discount Rate and Perceived Security:

Since security is a future good, people consume perceived discounted security, not riskiness and PTM as measured after the fact. As described in Sec. 6.1, and demonstrated mathematically in Sec. 6.5, a low discount rate makes some common kinds of investments look riskier to low discount rate persons, but illiquidity look less risky.

The Supply and Demand for Security Combined:

The demand curve for security shifts outwards with wealth or firm size. If security is a superior good, the demand curve may shift outwards quite rapidly.

Reduced supervision pushes the supply curve inwards. But better employees and tenants, lower leverage, risk-pooling and less risky activities push it outwards. Except perhaps for better employees, both pressures tend to reduce positive PTM. So the net effect isn't clear. But it seems plausible that supply at least doesn't fall.

So production and consumption increase with wealth or firm size, resulting in lower riskiness of income and profit, and, presumably, lower rates of employee turnover. Since lower riskiness also means PTM's closer to zero, rates of personal bankruptcy or firm failure fall. And if innovation requires a high positive PTM, innovation falls too.

But what about the implicit price of security in terms of other goods? Does the supply curve shift out slower than the demand curve, so that the implicit price rises, and resources transfer from producing more of other goods to producing more security? Or does the supply curve shift out faster than the demand curve, with the opposite result?

Considering the possibly large outward shift in the demand curve, and the conflicting pressures on the supply curve, the first possibility seems more likely.

But the question might prove hard to resolve empirically. For of course the same actions that reduce riskiness and bring PTM closer to zero also conserve scarce supervisorial time. How could one really tell if richer individuals and larger firm managers keep leverage lower and pool risks more than they would if wealth and firm size did not affect the demand for security?

The price and extent of insurance seem at first glance to offer some measure of the implicit price of security in terms of conventional goods. But even insurance also conserves time that might be spent keeping a closer watch on things. (Few losses stem purely from "acts of God". Hence the "moral hazard" to insurers: insurance makes losses more probable.)

In any case, it's certainly plausible that large firm managers do in fact divert considerable resources into producing "the quiet life" for themselves.

6.4 Riskiness and Proportional Third Moment

The riskiness and proportional third moment (PTM) of a distribution are closely but not rigidly related.

On the one hand, it's possible for people to have a very high PTM and yet keep their riskiness and downside exposure very low, by gambling for very high stakes at very long odds--for example, by buying lottery tickets.

On the other hand, for a *given* distribution, actions to reduce riskiness--like insuring, pooling, or reducing leverage--also bring PTM closer to zero. This is an improvement for negative PTM, but a loss for positive PTM.

These propositions can be demonstrated for a simple bimodal distribution.

Definitions:

The second moment, or standard deviation, of a probability distribution is the square root of probability-weighted squared deviations from the mean, or expected value, E. Denote it by σ.

The third moment is the cube root of probability-weighted cubed deviations from the mean. It measures the skew of a distribution, or relative length of the tails. It is zero for a symmetric distribution, positive for an upwards skew, and negative for a downwards skew. Denote it by σ_3.

The *riskiness* of a distribution is the standard deviation divided by the expected value: σ/E.

The *proportional third moment* or *PTM* is the third moment divided by the expected value: σ_3/E.

Properties of Proportional Third Moment and Riskiness:

Assume a bimodal, standardized distribution, so that expected value $E = 1$. So the standard deviation equals the riskiness and third moment equals the PTM.

The upper value, $U + 1$, occurs with probability p. $U \geq 0$ is the upward deviation from the mean.

The lower value, $1 - D$, occurs with probability $(1 - p)$, $0 \leq D \leq 1$. D is the downward deviation from the mean. $D \leq 1$ on the assumption that a person can lose at most all he has.

A little algebra yields the following relationships:

Expected value:

$$E = 1 = p(U + 1) + (1 - p)(1 - D)$$

Relationship of Upwards and Downwards Deviations

$$U/D = (1 - p)/p$$

Riskiness ($\sigma/E = \sigma$):

$$\sigma = (UD)^{1/2} = D[(1 - p)/p]^{1/2} = U[p/(1 - p)]^{1/2}$$

Proportional Third Moment ($\sigma_3/E = \sigma_3$)

$$\sigma_3 = [UD(U - D)]^{1/3} = D[(1 - p)(1 - 2p)/p^2]^{1/3}$$
$$= U[p(1 - 2p)/(1 - p)^2]^{1/3}$$

Proportional Third Moment in terms of Riskiness, σ

$$\sigma_3 = \sigma [\sigma/D - D/\sigma]^{1/3} = \sigma [U/\sigma - \sigma/U]^{1/3}$$
$$= \sigma (1 - 2p)^{1/3}/[p(1 - p)]^{1/6}$$

A number of points are apparent on inspection, or easily derived:

1. The PTM = 0 for $U = D = \sigma$; $p = 1/2$, --a symmetric distribution. For $U > D$, $p < 1/2$, the PTM is positive; for $U < D$, $p > 1/2$, it is negative.

2. Assuming $D \leq 1$, the PTM reaches a minimum value of $-(1/4)^{1/3}$ = (approx) $-.63$ when $U = 1/2$, $D = 1$, $p = 2/3$, and $\sigma = (1/2)^{1/2}$.

3. There is no maximum PTM, even holding σ constant. For a given σ, the PTM will be higher for higher U, smaller p and smaller D. For very long odds, the formula for the PTM becomes approximately: $\sigma_3 = \sigma\, p^{-1/6}$. In other words, it depends only on riskiness and probability of U.

Effect on PTM of Measures to Reduce Riskiness:

Consider three possible actions: a. insuring, b. pooling, and c. reducing leverage.

a. Insuring:

Imagine the bimodal distribution is "insured" by a payment of $Z \leq D$, contingent on loss D. The payment to the insurer must be $Z(1 - p)/p + M$, where M is a risk premium, assumed small.

Then:

Expected Value:

$$E = 1 - M = p[1 + U - Z(1 - p)/p - M] + (1 - p)[1 - D + Z - M]$$

Riskiness:

$$\sigma /E = (D - Z)[(1 - p)/p]^{1/2}/(1 - M)$$

PTM:

$$\sigma_3/E = (D - Z)[(1 - p)(1 - 2p)/p^2]^{1/3}/ (1 - M)$$

Obviously, provided M is relatively small, riskiness falls and PTM shrinks towards zero as the amount of insurance, Z, increases.

b. Pooling:

Another way to reduce riskiness is to pool investments.

Suppose we pool n of the bimodal distributions, assumed independent. Then:

<u>Riskiness:</u>

$$\sigma / E \quad = \quad D[(1 - p)/np]^{1/2}$$

<u>PTM:</u>

$$\sigma_3/E \quad = \quad D[(1 - p)(1 - 2p)/n^2 p^2]^{1/3}$$

As n increases, the PTM actually approaches zero faster than riskiness falls, --by the 2/3 power of n instead of the 1/2 power.

<u>c. Reducing Leverage</u>

Imagine the bimodal distribution is leveraged by an amount V , $0 \leq V \leq 1$. So the expected value E becomes 1 - V. Then:

<u>Riskiness:</u>

$$\sigma / E \quad = \quad \sigma / (1 - V) \quad = \quad D[(1 - p)/p]^{1/2}/(1 - V)$$

<u>PTM:</u>

$$\sigma_3/E \quad = \quad \sigma_3/(1 - V) \quad = \quad D[(1 - p)(1 - 2p)/p^2]^{1/3}/(1 - V)$$

So the lower the leverage, the lower the riskiness, and the closer the PTM to zero.

6.5 Discount Rate and Perceived Riskiness, PTM, and PV^D

Case 1: Receipt Before Liability:

Suppose a person wants to borrow money to make an investment. He expects the investment to pay off with some probability before the repayment date of the loan. If the investment fails, he will have a liability at that time.

In this case, the investment appears less risky to a relatively high discount rate person, because such a person gives relatively more weight to the near receipt than the remote liability. In fact under some circumstances, a higher discount rate person actually accords a higher risk-free present value to such an investment.

The same receipt-before-liability pattern applies to theft, suggesting that high discount rate persons are more likely to steal than low discount rate persons.

Here's a simple way to model receipt before liability:

Imagine a bimodal distribution with a payment of X due with probability p at time t_1, and a liability, Y, due with a probability (1-p) at time $t_2 > t_1$. The discount rate is r. (If A is the amount of payoff, and i is the bank's interest rate, then $X = A - Ye^{-i(t_2-t_1)}$.)

Then the expected present value, riskiness, and proportional third moment (PTM) are:

$$(5.1) \qquad E = pXe^{-rt_1} - (1-p)Ye^{-rt_2} > 0$$

$$(5.2) \qquad \sigma/E = \left[\frac{Xe^{-rt_1} + Ye^{-rt_2}}{pXe^{-rt_1} - (1-p)Ye^{-rt_2}}\right] [p(1-p)]^{1/2}$$

$$(5.3) \qquad \sigma_3/E = \left[\frac{Xe^{-rt_1} + Ye^{-rt_2}}{pXe^{-rt_1} - (1-p)Ye^{-rt_2}}\right] [p(1-p)(1-2p)]^{1/3}$$

$$> 0 \text{ for } p < 1/2$$

An increase in the discount rate, r, affects expected present value, riskiness, and PTM as follows:

$$(5.4) \qquad \frac{dE}{dr} = -t_1 pXe^{-rt_1} + t_2(1-p)Ye^{-rt_2}$$

If the second term is large enough, present value may actually increase as the discount rate rises. So if poorer people have higher discount rates, this investment may look more valuable to a poorer than to a richer person.

$$(5.5) \qquad \frac{d}{dr}\left[\frac{Xe^{-rt_1} + Ye^{-rt_2}}{pXe^{-rt_1} - (1-p)Ye^{-rt_2}}\right] = \frac{XYe^{-r(t_1+t_2)}(t_1 - t_2)}{(pXe^{-rt_1} - (1-p)Ye^{-rt_2})^2}$$

$$< 0 \text{ since } t_2 > t_1$$

So riskiness falls and PTM gets closer to 0 as the discount rate increases. If poorer people have a higher discount rate, they perceive such an investment as less risky.

Case 2: Riskiness Increases with Distance in the Future:

Many investments deliver not one but a series of payoffs. The more remote the payoff, the greater the uncertainty of its amount. For some sorts of investments like bonds uncertainty (including interest rate uncertainty) increases only slowly and slightly over time. For other sorts of investments, like used cars, uncertainty increases rapidly.

The higher a person's discount rate, the less risky an investment appears whose payoffs increase in uncertainty over time. Consequently, an investment whose payoffs increase rapidly in uncertainty may actually look more valuable to a high rather than a low discount rate person. So here's another reason apart from high maintenance requirements why poorer people and smaller firms may prefer used equipment.

Here's a simple way to model an investment whose payoffs increase in uncertainty over time:

Imgine an investment with two positive payoffs at different times in the future: X at time t_1, and Y at time t_2, $t_2 > t_1$.

The standard deviation for X is σ_X, for Y is σ_Y, and the covariance is σ_{XY}. Assume the riskiness of X is less than that of Y: $\sigma_X/X < \sigma_Y/Y$.

Let:

$A = Xe^{-rt_1}$ be the present value of X

$B = Ye^{-rt_2}$ be the present value of Y

$\sigma_A = \sigma_X e^{-rt_1}$ be the present value of σ_X

$\sigma_B = \sigma_Y e^{-rt_2}$ be the present value of σ_Y

$\sigma_{AB} = \sigma_{XY} e^{-r(t_1+t_2)}$ be the present value of σ_{XY}

$R_A = \frac{\sigma_A}{A} = \frac{\sigma_X}{X}$ be the riskiness of A and X

$R_B = \frac{\sigma_B}{B} = \frac{\sigma_Y}{Y}$ be the riskiness of B and Y

$C_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$ be the corellation coefficient

) do not depend on r

$k = \frac{A}{B} = \frac{X}{Y} e^{-r(t_1-t_2)}$ be the ratio of A to B

An increase in the discount rate, r, raises k, the ratio of A to B.

$$\frac{dk}{dr} = \frac{X}{Y} e^{r(t_2-t_1)} (t_2 - t_1) > 0 \tag{5.6}$$

So if poorer persons have a higher discount rate, they set a proportionally higher value on the nearer payoff.

The riskiness of the present value of the investment is:

$$R = \frac{(R_A{}^2A^2 + r_B{}^2B^2 + 2C_{AB}R_AR_B)^{1/2}}{A + B} \tag{5.7}$$

Substituting $k = A/B$:

$$(5.8) \qquad R = \frac{(k^2R_A{}^2 + R_B{}^2 + 2kC_{AB}R_{AB})^{1/2}}{1 + k}$$

So that:

$$(5.9) \qquad \frac{dR}{dk} = \frac{1}{R(1+k)^3}[\ (kR_A + R_B)(R_A - R_B) + (1-k)R_AR_B(C_{AB}-1)\ ]$$

$$< 0 \quad \text{(usually)}$$

Riskiness falls as $k = A/B$ rises for $C_{AB} = 1$ -- perfect positive corellation. In the worst case, where $C_{AB} = -1$, riskiness falls provided $k < R_B/R_A$ or $\sigma_A < \sigma_B$. So in general, riskiness falls as discount rate rises where the second payoff is considerably more risky than the first, and/or payoffs are positively corellated.

Case 3: Riskiness in the Timing of Payoff--Liquidity:

A liquid asset is one that can be sold very quickly for a fairly certain market price. The price for an illiquid asset is less certain, because the market is "thin". To realize a good price may require a considerable wait for the right buyer to come along.

The riskiness of illiquidity may appear less to richer, low discount rate persons. For such persons can better afford to wait for a good price. So richer people may enjoy a comparative advantage in owning illiquid assets, such as Old Masters, or large undeveloped parcels in the suburbs.

Here's a simple way to model an illiquid asset:

Suppose an asset can be sold for \$1 with equal probability anytime between now and time T. The longer T, the more illiquid the asset.

The expected present value of the asset is:

$$(5.10) \qquad E = \frac{1 - e^{-rT}}{rT}$$

The riskiness of the asset is:

$$(5.11) \qquad \sigma/E = \left[\frac{rT}{2} \frac{(1+e^{-rT})}{(1-e^{-rT})} - 1 \right]^{1/2}$$

And the proportional third moment of the asset is:

$$(5.12) \qquad \sigma_3/E = \left[-2 + \frac{3rT}{2} \frac{(1+e^{-rT})}{(1-e^{-rT})} - \frac{(rT)^2}{3} \frac{(1+e^{-rT}+e^{-2rT})}{(1-e^{-rT})^2} \right] < 0$$

It's apparent from inspecting these expressions that as rT rises --due to a rise in discount rate r, or lengthening of time T--riskiness increases and proportional third moment becomes more negative. So the higher a person's discount rate, or the more illiquid the asset, the less desirable it looks.

6.6 Leverage, Wealth, and Riskiness

If leverage--operating or financial--falls with firm size, so does the riskiness of profit.

Notation:

Y -- gross income of firm, rising with firm size

σ_Y -- standard deviation of gross income

σ_Y/Y -- riskiness of gross income

C -- total operating costs, fixed and variable

σ_C -- standard deviation of operating costs

σ_C/C -- riskiness of operating costs

σ_{YC} -- covariance of gross income and operating costs

σ_{YC}/YC -- corellation of gross income and operating costs

D -- debt service, assumed invariant

$P = Y - C - D$ -- profit

$PS = P/Y$ -- profit share

$\sigma_P = (\sigma_Y{}^2 + \sigma_C{}^2 + 2\sigma_{YC})^{1/2}$ -- standard deviation of profit

$\sigma_P/P = \dfrac{\left(\frac{\sigma_Y{}^2}{Y^2} + \frac{\sigma_C{}^2}{Y^2} + \frac{2\sigma_{YC}}{Y^2} \right)^{1/2}}{PS}$ -- riskiness of profit

$\dfrac{Y}{Y - C} = \dfrac{1}{1 - C/Y}$ -- operating leverage

$\dfrac{Y}{Y - D} = \dfrac{1}{1 - D/Y}$ -- financial leverage

$\dfrac{Y}{Y-C-D} = \dfrac{Y}{P} = \dfrac{1}{PS}$ -- total leverage

Assumptions:

1. Operating, financial, and total leverage fall as firm size increases. So C/Y, D/Y, and $(C+D)/Y$ fall, while PS rises.

2. Riskiness of gross income, σ_Y/Y falls or does not increase with firm size, due to economies of scale in risk-pooling.

3. Riskiness of operating costs, σ_C/C falls or does not increase with firm size. Presumably economies of scale in risk-pooling affect operating costs too. Moreover, the larger a firm, the larger the fixed portion of operating costs, (eg. "overhead"), and so the smaller the possible fluctuations.

4. The covariance of gross income and operating costs, σ_{YC}, is small compared to the variances, σ_Y^2 and σ_C^2, and the corellation, σ_{YC}/YC, does not increase too much with firm size. (Covariance and corellation are probably > 0. If the market price of output unexpectedly rises, so that gross revenue rises, operating costs may rise too, as the firm increases output. If variable (marginal) costs unexpectedly increase, the firm may raise the price (if it can). It may also cut output--producing a negative covariance.) Assuming a relatively small covariance and corellation means assuming the firm has only limited ability to respond to random fluctuations.

Consequences:

The riskiness of profit can be written:

$$(6.1) \qquad \frac{\sigma_P}{P} = \frac{\left[\frac{\sigma_Y^2}{Y^2} + \frac{\sigma_C^2}{C^2}\frac{C^2}{Y^2} + \frac{2\sigma_{YC}}{YC}\frac{C}{Y} \right]^{1/2}}{PS}$$

The denominator increases with firm size by assumption 1. The first two terms in the numerator fall with firm size, by assumptions 1, 2, and 3. The third term of the numerator probably rises with firm size. But by assumption 4, it is relatively unimportant.

So riskiness of profit falls as firm size increases.

6.7 Riskiness and Risk Premium

Assuming that consumption of security rises with wealth or firm size, what happens to the risk premium--the difference between measured rate of discount and true, risk-free rate of discount?

For a simple set of assumptions, it's easy to show that risk premium falls unless the cost of providing security rises dramatically with wealth.

Notation:

R -- riskiness

Q -- all goods besides riskiness

p -- (negative) price of riskiness in terms of other goods

r -- true discount rate

r_m -- measured discount rate

$r^* = r_m - r$ -- risk premium

Demonstration:

Suppose income y is the value of all consumption, including the consumption of the negative good, riskiness.

(7.1) $$y = Q - pR > 0$$

Assuming $1/r$ is large compared to 1, wealth W is:

(7.2) $$W = \frac{y}{r} = \frac{Q - pR}{r}$$

Measured discount rate, r_m is:

(7.3) $$r_m = \frac{Q}{W} = \frac{1}{1 - \frac{pR}{Q}} r$$

pR/Q measures the ratio of the value of riskiness consumed to the value of everything else consumed. By assumption, consumption of riskiness, R, falls with wealth, and consumption of everything else, Q rises without limit. So the price of riskiness, p, must rise very rapidly to make pR/Q rise. Since r falls with wealth, r_m falls too unless p rises very rapidly.

Risk premium, r*, is:

$$(7.4) \qquad r^* = r_m - r = \frac{pR}{Q} r_m = \frac{\frac{pR}{Q}}{1 - \frac{pR}{Q}} r$$

So if pR/Q falls, risk premium falls too.