Wealth Inequality, Working Capital, Growth and Instability: A Dual Economy Simulation Model

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Small businesses generate half of employment in the US. They rely heavily on bank lending to maintain cash flow, and rely primarily on real estate for loan collateral. They were hit heavily in 2008; recovery has been slow and partial. Meanwhile, large businesses and banks hoard capital. To interpret the small business problem, I simulate a "dual" economy: In a "Rich" sector, a small population owns most of the land and much of the physical working capital—food and other supplies—call it "food". Food serves both as consumption good and capital stock. A "Poor" sector, with most of the population, borrows working capital—food—from the Rich sector across a barrier of transaction costs, returning it with interest, using its own land and food stock as collateral. Starting from a very small food inventory, the economy grows for a hundred periods. The model shows the following: In Rich compared to Poor, wages are higher and interest rates are lower; output per man-hour is higher and output per acre is lower. This unequal economy is less productive and slower growing than an identical equal economy. In addition, small shocks in total factor productivity can generate large fluctuations, which reduce overall productivity and growth, and fall largely on Poor.

Small businesses (officially defined as having under 500 employees) employ about half of all employees. They consist of about 23 million sole proprietorships, and 5.7 million with employees. Since 1995, they have created two out of every three net new jobs. According to a recent SBA survey, they rely heavily on bank lending to maintain cash flow, and depend primarily on real estate for loan collateral. However, they were hit much harder than large businesses during the Great Recession and have been slow to recover. This leads former Small Business Administration Administrator, Karen Gordon Mills, to pose the question: "Is there a credit gap in small business lending?" She finds small businesses were hit harder, not only due to a loss of sales, but to their heavy reliance on bank capital, and the lost value of loan collateral, primarily real estate. Meanwhile, banks have severely cut back small business lending, both due to increased caution following the Recession, and as part of a long-term trend to larger banks, which don't find small loans profitable enough to compensate for higher administrative costs.. (Mills & McCarthy, 2014).

The split between large and small businesses, like the related split between the One Percent and the Ninety-nine Percent, makes the U.S. seem increasingly like a dual economy. In fact, economic historian Peter Temin has just published a working paper entitled, "The American

Dual Economy: Race, Globalization, and the Politics of Exclusion."(Temin, 2015) Dualism isn't a new idea. British Prime Minister and novelist Benjamin Disraeli described Britain as "two nations," "the Privileged and the People." Temin draws on the 1954 development model of W. Arthur Lewis, who divided underdeveloped economies into a high wage "capitalist" sector and a low wage "subsistence" sector, with a transaction cost barrier limiting workers' movement from subsistence. Temin names his sectors Finance, Technology and Electronics (FTE) and "low-wage", again with transaction cost barriers limiting movement. Institutional economist Robert Averitt's 1968 book, *The Dual Economy: the Dynamics of American Industry Structure*, identified systematic differences in factor proportions and prices between large and small firms—what he called the "core" economy and the "peripheral" economy (Averitt, 1968). However, instead of seeing these differences as connected to underlying wealth inequality, Averitt saw big firms simply as superior to small ones—a prejudice that continues to this day.

In *Consequences and Causes of Unequal Distribution of Wealth* (Cleveland, 1984) , I modelled unequal wealth distribution between individuals or firms as arising from unequal land ownership. The "Large" sector hires labor from or rents land to the "Small" sector across a barrier of transaction costs. Simply due to different factor proportions, Large enjoys a lower cost of capital and a higher cost of labor than Small, that is Large pays or imputes a lower discount rate and a higher wage. Large obtains more output per unit of labor and less per equivalent unit of land—a difference obscured by Large's comparative advantage in holding higher quality land. Other results also follow from comparative advantage. For example, Large employs higher quality workers as a way of extending its scarce labor supply. The greater the inequality between Large and Small and/or the higher the transaction cost barrier, the less productive the economy.

In the present paper, I present a simulation growth model further exploring differences between Large and Small, which I rename Rich and Poor. Rich, with a tenth of the population, owns most of the assets, including 90% of the land and much of the physical working capital—food and other supplies—call it "food". Food serves as consumption good while food inventory serves as working capital stock. Poor, with 90% of the population and 10% of the land, borrows working capital—food—from Rich across a barrier of transaction costs, returning it with interest, with its own land and food stock as collateral. Starting from a very small food inventory, the economy grows for a hundred periods. The model shows the following: As expected from factor proportions, in Rich compared to Poor, wages are higher and interest rates are lower; output per man-hour is higher and output per acre is lower. This unequal economy is less productive and slower growing than an identical equal economy. In addition, small shocks in total factor productivity can generate large fluctuations, which fall almost entirely on Poor, and reduce overall productivity and growth.

This model relies on land as a primary form of capital along with physical working capital. In so doing, it follows David Ricardo's classic "corn" model, which Ricardo computed numerically in Chapter 6 of his *Principles* (Ricardo, [1818] 1996). While Ricardo's model nominally represents an agricultural cycle in which farmers rely on an inventory of grain—"corn" in British—to survive from one harvest to the next, it is clearly intended to represent the entire economy. Using land as a factor of production breaks from the elegant Solow growth model (Solow, 1956), still alive and well after almost fifty years, for example in Thomas Piketty's *Capital in the Twenty-First Century* (Piketty, 2014). Solow's model assumes that all capital depreciates and must be replaced; in the real world, land not only lasts forever, but appreciates in a bubble. In fact, as we were painfully reminded in 2008, land drives the business cycle. Thus economists like Charles Calomiris recommend legislation to limit banks' ability to lend against real estate (Calomiris, 2015).

A Brief Digression on Classical versus Neoclassical Terminology

How can one write a paper about concepts that no longer exist in modern neoclassical economics?

The classical economists recognized three factors of production, land, labor, and capital. Land meant all natural resources, not just farmland. Labor meant labor services. Capital meant physical man-made things used in production. The term wealth, as in *Wealth of Nations*, meant all useful man-made things, so that capital was a subset of wealth. The classicals recognized two types of capital: "fixed" capital, like buildings and machines, and "circulating" capital, inventories of finished goods or goods in process. Land differed from capital in not being man-made, and in being permanent instead of depreciating and needing to be replaced. The king granted and protected titles to land; capital belonged to those who made it. Landowners received "rent", which was unearned income based on the superiority of their land to marginal land.

Capital owners received interest or profit; the classicals never quite figured out the relationship. And of course workers earned wages for their labor.

Neoclassical economics merges land and capital into one great blob of things that produce output when combined with labor. Gone is the distinction between permanent and depreciating assets, and between earned and unearned income. Concepts like “financial capital” further confuse the issue.

In this paper I focus on what the classicals called “circulating capital”, short-lived physical inventories of finished goods and goods in process. Such capital would include for example, oil in tanks, but not oil in the ground. The term “circulating” perfectly conveys the idea that these goods are both normally transported from one place to another, and are quickly replaced.

For my purposes here, the closest I can get is “physical working capital.” Think of it from the perspective of a small grocer. Every month he borrows on his line of credit to purchase stock and to pay workers, who will immediately purchase consumption goods, maybe from the same store! At the end of the month, he repays the credit from sales, with interest. If he can’t get credit, he’s out of business. We’re talking short-lived physical goods which will quickly be consumed and replaced.

1. Simple simulation model with land, labor and working capital—“food”.

See Appendix A for complete description with equations.

I develop a simple simulation model with uniform land and population of simple “workers” who produce a single good, “Food”. Food serves both as a consumption good and a storable stock of physical working capital. The model runs over an arbitrary hundred periods, during which working capital – “Food” – accumulates. This is how the model economy grows. Food stock essentially serves to bridge the time between start and completion of production each period. A larger stock to feed the workers, subject to diminishing returns, allows greater production.

The workers own both land and working capital. Hence they receive both wages and profit. This profit is equivalent to rent for the land. The labor supply function (see Appendix A) is affected by both wage and rent income; the higher the rent the higher the wage to elicit a given supply of labor. See Figure A.1. Obviously, rent and wage rates must be imputed since workers receive them as a lump.

Because there is time in this model, there must be a discount rate used to make trade-offs between past and present, as well as to place a value on land. I use a discount rate that depends on an exponential of the ratio of expected expenditure on food during the production period to food stock at the beginning. The higher the ratio the higher the discount rate. See Figure A.2. This is quite different from the usual concept of discount rate as reflecting an agent's internal time preferences.

Since this is a multi-period model, land value must include projections for the future. As explained in Appendix A, the expectation of future appreciation reduces the discount by a growth factor. I estimate this growth factor by taking a running average of prior growth.

Assume a well-behaved production function, depending on land and labor, linear homogeneous (although that is not necessary) –but not Cobb-Douglas. The production function implicitly depends on the working capital ("food") necessary to pay workers while production proceeds. All factors – labor, land, and working capital – receive their marginal products. I use a "Keynesian" consumption function: consumption depends primarily (85%) on current income and secondarily (4%) on "wealth" in the form of accumulated capital stock and projected land value.

Because it depends on crude projections, the model has built-in instability—a tendency to overshoot and then undershoot. A small shock such as a one-period blip in total factor productivity may induce a large bounce.

2. Simple simulation model with Rich and Poor sectors.

I run this model with two separate "sectors" of an economy, Rich with 9/10 of the land and 1/10 the population, and Poor with 1/10 the land and 9/10 the population. As the model runs, both regions accumulate food capital, Rich more than Poor, but at a slowing rate, up towards a final equilibrium. Wages rise and interest rates fall in both regions, but Rich wages always exceed Poor wages, while Rich discount rates are always lower than Poor rates.

I allow Rich to loan Food to Poor as the economy grows, advancing it at the beginning of each period and recovering it with interest at the end. Rich lending is restricted, as in the real world, by transaction costs, moral hazard and adverse selection.

Figure 1, Output With Equality, shows what the output of this model economy would be if the two sectors were equal. Total output (green) grows to something over 1000 units. By contrast, ***Figure 3 Output with Inequality***, shows output of the two sectors together with total output. Rich (red) has output a bit over 100, Poor (blue) a bit over 400, for a total (green) between 500 and 600. So the unequal economy produces about half as much as the equal economy.

Figure 2, Capital Accumulation with Equality, shows capital stock accumulation (green) in the equal economy, reaching 3000 units and still growing. ***Figure 4, Capital Accumulation with Inequality***, shows Rich (red) accumulating about 350 units, Poor (blue) something over 200. Combined capital stock accumulation (green) in the unequal economy reaches a little under 600 units and apparently levels off. So inequality in this model affects growth even more drastically than output. The rising yellow line represents lending from Rich to Poor. The most striking feature of Figure 4 is that after a brief spurt of capital accumulation, Poor begins to decumulate capital, relying instead on borrowing from Rich. That is, as soon as Rich has accumulated enough capital, it becomes cheaper for Poor to borrow rather than accumulate its own capital. If lending is not allowed, Poor will continue to accumulate capital through the run.

Figure 5, Consumption with Inequality, shows that Poor, with 9/10 the population, supplies consumes about 4 ½ times as much as Rich. ***Figure 6, Saving with Inequality***, shows that both Rich and Poor save at high rates at the start, but rates fall rapidly and soon go negative for Poor – which is consistent with Poor's dis-saving and reliance on lending from Rich.

Figure 7, Labor Supply with Inequality, shows that Poor, with 9/10 the population, applies about six times the labor as Rich. ***Figure 12, Labor Supplied per Capita with Inequality***, shows that Rich workers work almost twice as much as Poor workers, a reflection of the higher wage.

Figure 8, Profit, Interest and Wage Income with Inequality, shows the breakdown of earnings between Rich and Poor. Poor earns wages and profit, while Rich earns wages, profit and interest on its loan. Poor wages, for 9/10 the population, are about 2 ½ times those of Rich. Poor profit, however is over 25 times Rich profit, because it represents a return to scarce land.

Figure 9, Wage Rate with Inequality, shows Rich with about double the wage rate of Poor. This of course reflects the relative scarcity of labor in the Rich sector. ***Figure 10, Profit Rate with Inequality***, shows the dramatic difference between the profit rate or rent per acre of Poor and Rich—over 200 to one at the end of the run. High rent rate, which equals the marginal product of land, reflects the relative scarcity of land in the Poor sector.

Figure 11, Discount and Interest Rates with Inequality, shows how discount rates fall for both Rich and Poor over the run, with the rate for Poor always a bit higher, ending up around 10% for Rich and 11% for Poor. (If the economy were equal the final discount rate would be about 6%.) If lending were not permitted, the discount rate gap between Rich and Poor would be much greater. If there were no transaction costs in lending, Rich and Poor would have the same discount rates. It makes sense to speak of imputed internal discount rates when Rich cannot lend to Poor. Once Rich lends to Poor, there are actual market interest rates, as supply meets demand for capital.

Figure 13, Output Per Capita with Inequality, shows how Rich produces over twice as much per capita as Poor. ***Figure 14, Labor Productivity = Output per Man-Hour with Inequality,*** shows how Rich still gets higher productivity per man-hour than Poor, though the difference is less dramatic – because Rich works longer hours.

Figure 15, Capital Productivity = Output/Stock with Inequality, shows how capital stock is always more productive in Poor than in Rich. Another word for capital productivity is capital turnover. In Poor, capital productivity first declines and then rises again as Poor begins to borrow from Rich. If there is no borrowing, then capital productivity declines for both Rich and Poor.

Figure 16, Land Productivity = Output per Acre with Inequality, shows that Poor has vastly higher land productivity than Rich (some 36 times), simply because Poor has so much less land per capita.

Figure 17, Land Value with Inequality, shows how Poor, with only 1/10 the land area, has dramatically higher land value, (over 20 times). Land value per acre (not shown) is over 200 times greater. Land value does not precisely track rent, shown in ***Figure 10***, because of the expected growth factor when rent is capitalized. In particular, land value in Rich shows a downward trend and then a kink up at the beginning of the run. This happens because expected growth is high at the beginning of the run, reducing the effective discount rate, and thus raising land values.

Figure 18, Wealth = Land Value + Capital Stock with Inequality, shows how Poor also holds most of the economy's wealth, about four times as much as Rich. However, Rich has over twice the wealth per capita of Poor (not shown). (Recall that consumption is proportional to wealth in the model, with a small factor.)

3. Simulation Model with a Shock

Figure 19, Output with Shock, with Equality, shows the effect of a one-period positive shock halfway through the run. That is, total factor productivity increases dramatically for just one period. As a result, output declines for over 10 periods after the shock, and has still not quite recovered by the end of the run. ***Figure 20, Capital Stock with Shock, with Equality,*** shows why: excessive consumption in response to the positive shock causes a plunge in capital stock. The plunge happens partly because higher wealth increases consumption, and partly because agents naïvely project past events into the future.

Figure 21, Output with Shock, with Inequality, shows the effect on Rich and Poor of a one-period positive shock halfway through the run. Again, output declines for over 10 periods after the shock, and has still not quite recovered by the end of the run. ***Figure 22, Capital Stock with Shock, with Inequality,*** shows why: excessive consumption in response to the positive shock causes a plunge in capital stock. But in this case, most of the plunge happens in Poor. Rich is barely affected, except that it sharply increases lending to Poor.

Figure 27, Labor Supply with Shock, with Inequality, Figure 28 Consumption with Shock, with Inequality and ***Figure 29, Profit, Interest, and Wage Income with Shock, with Inequality*** show how consumption, labor supply and wages fall after the shock.

Figure 30, Saving with Shock, with Inequality, shows how there's actually an echo of the shock some 10 periods out, with a second spike in saving, and a corresponding dip in consumption.

4. Discussion

This simple model of a dual economy challenges conventional macroeconomics in at least three ways:

First, it shows how wealth inequality can dramatically limit productivity and growth (hardly news to a development economist). It also demonstrates how inequality affects factor proportions and prices in the rich and poor parts of an economy. Thus, most obviously, labor productivity is relatively high in rich parts, while land and working capital productivity are relatively high in poor parts. This difference turns up in U. S. Census statistics on large and small firms. For example, according to 2007 Census data, comparing firms with under half a million in sales to those with over $100 million, the small firms averaged 15 employees per $100 million sales while the big ones averaged only three.

Second, the model highlights the key role of physical working capital in boom and bust. In this model, it is clearly the loss of working capital, not a lack of aggregate demand, that that prolongs a slump after a shock. In an unequal economy, the burden of this loss falls hardest on the poor sector.

Third, the model ties the discount rate to a physical base independent of money: the ratio of ***needed*** to ***available*** working capital. Needed working capital means the amount necessary to survive until more comes in, usually over a period of production. Available working capital means the amount in stock at the beginning of the period, and/or accessible by borrowing. This formulation would make perfect sense in a third-world country, particularly in the face of a famine.

Images.

Appendix A.

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Fig. 1. Output with Equality

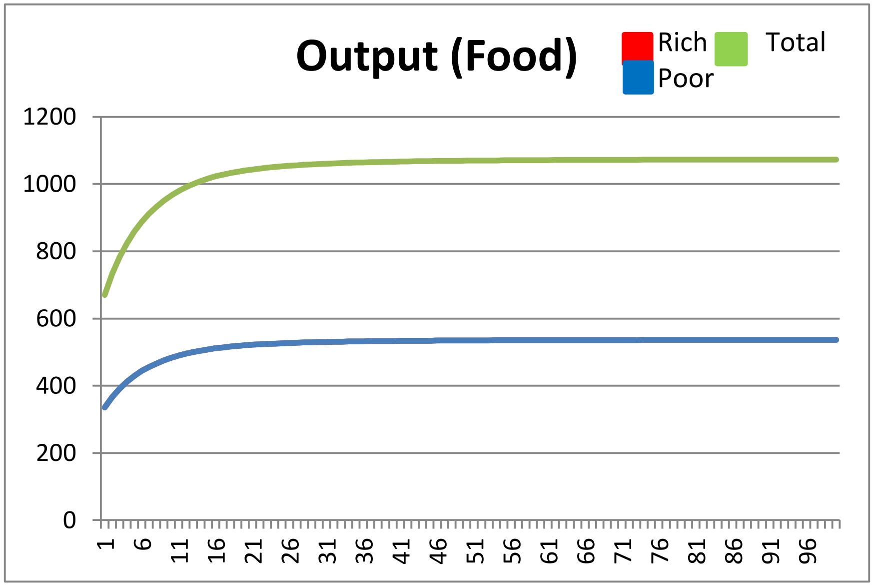


Fig 2. Capital Accumulation with Equality

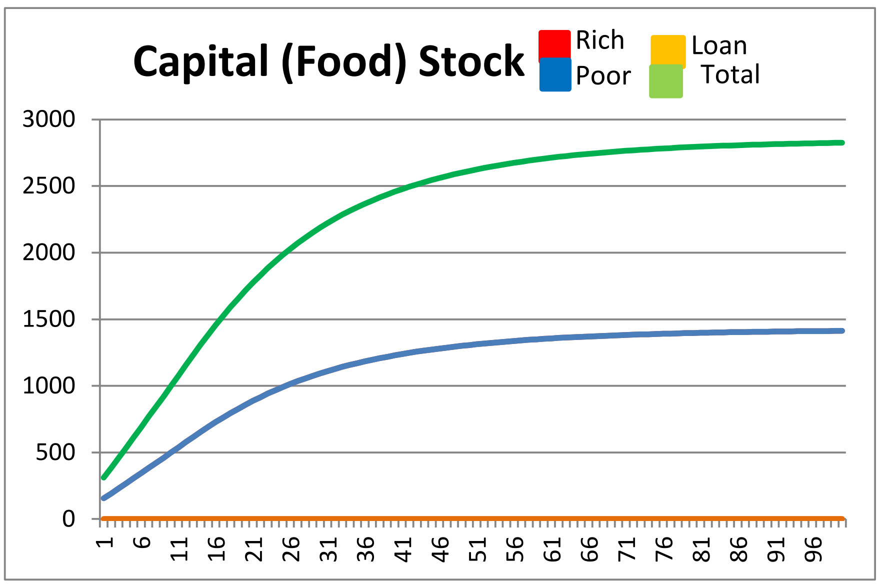


Fig. 3. Output with Inequality

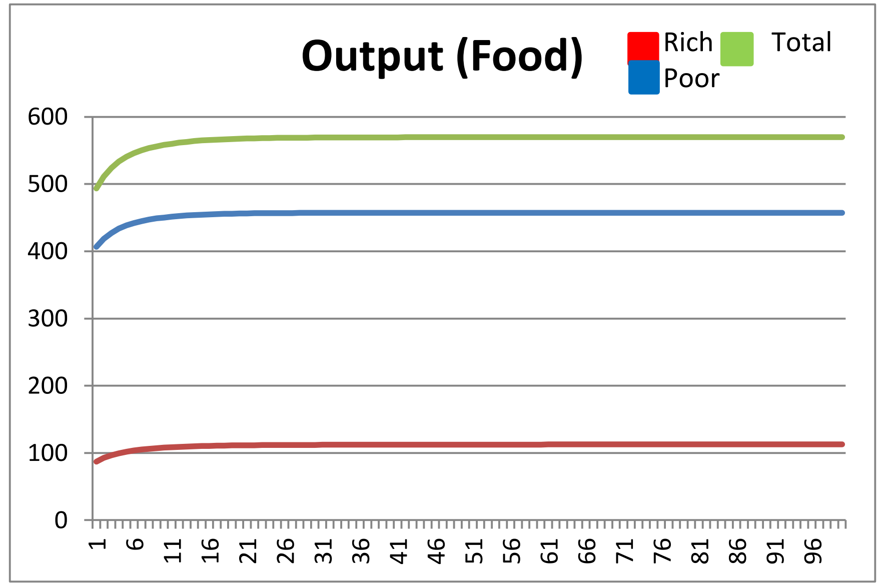


Fig. 4. Capital Accumulation with Inequality

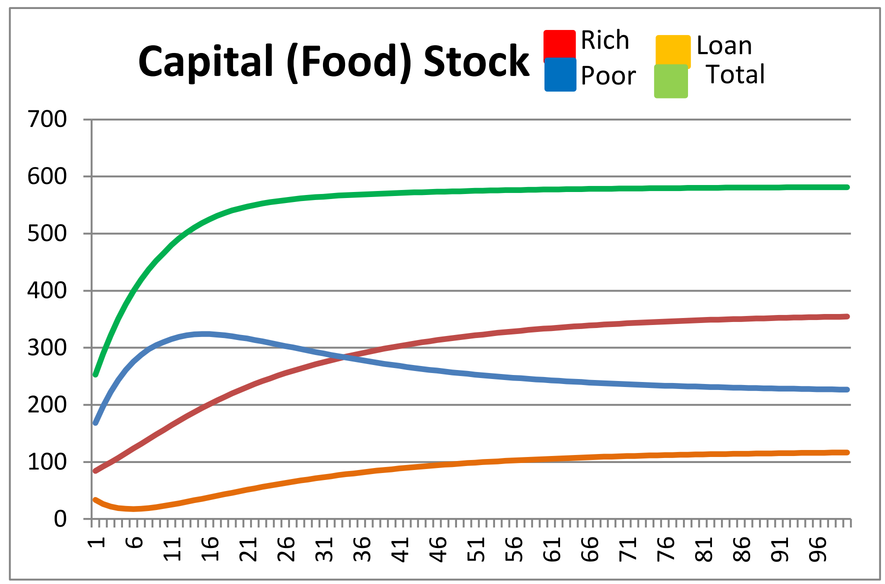


Fig. 5. Consumption with Inequality

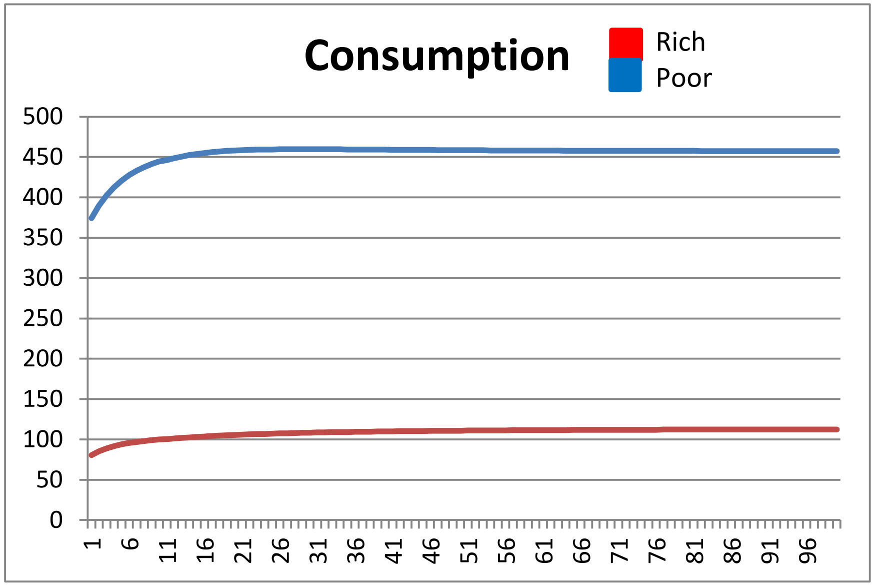


Fig. 7, Labor Supply with Inequality

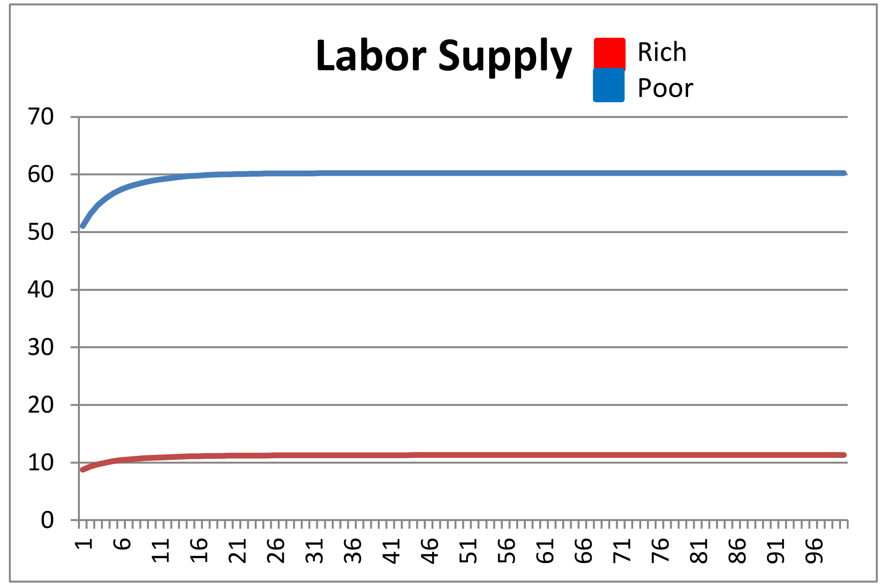


Fig. 6. Saving with Inequality

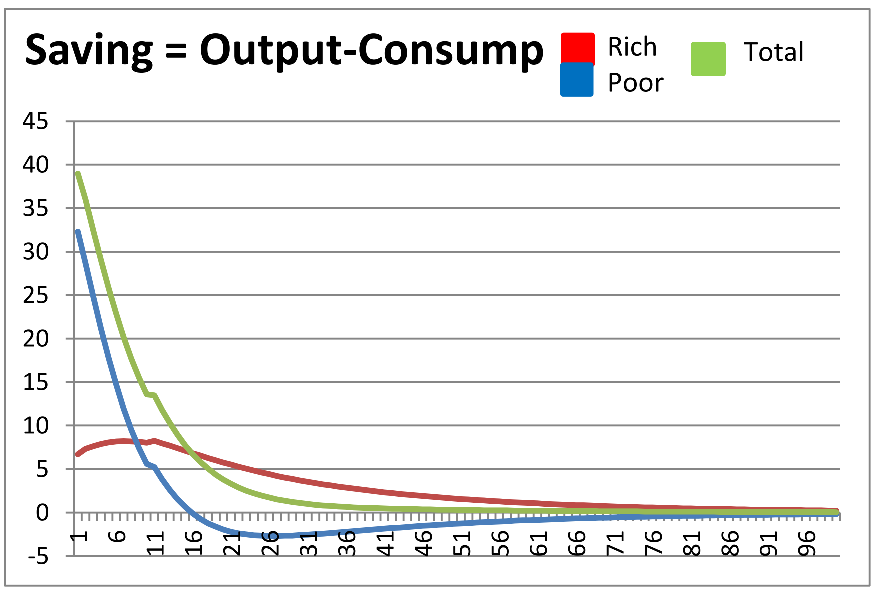


Fig. 8. Profit, Interest and Wage Income with Inequality

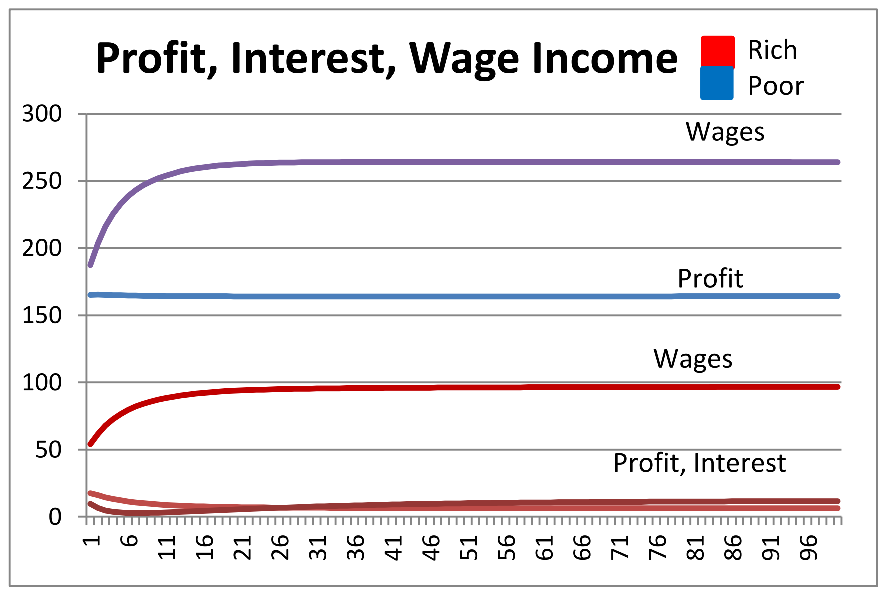


Fig. 9. Wage Rate with Inequality

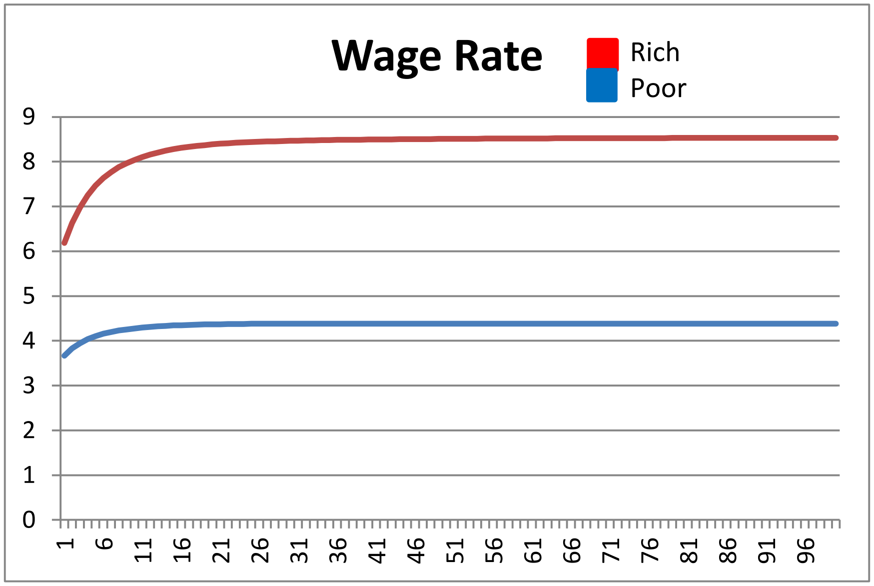


Fig. 10. Profit Rate with Inequality

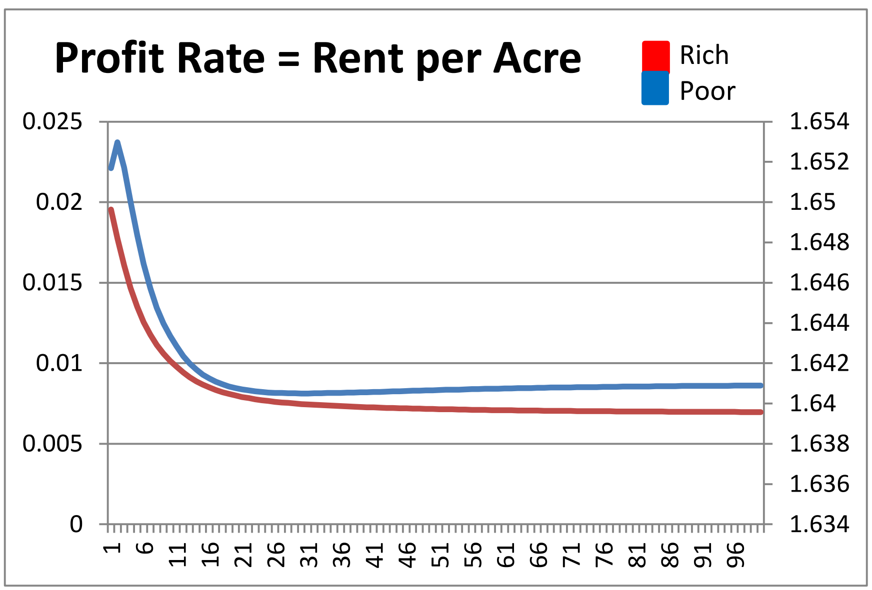


Fig. 11. Discount and Interest Rates with Inequality

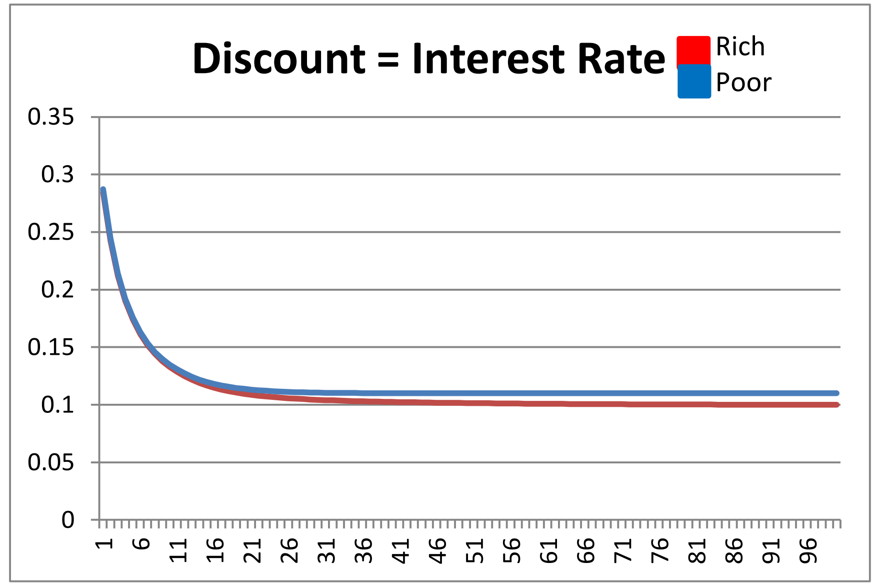


Fig. 12. Labor Supplied per Capita with Inequality

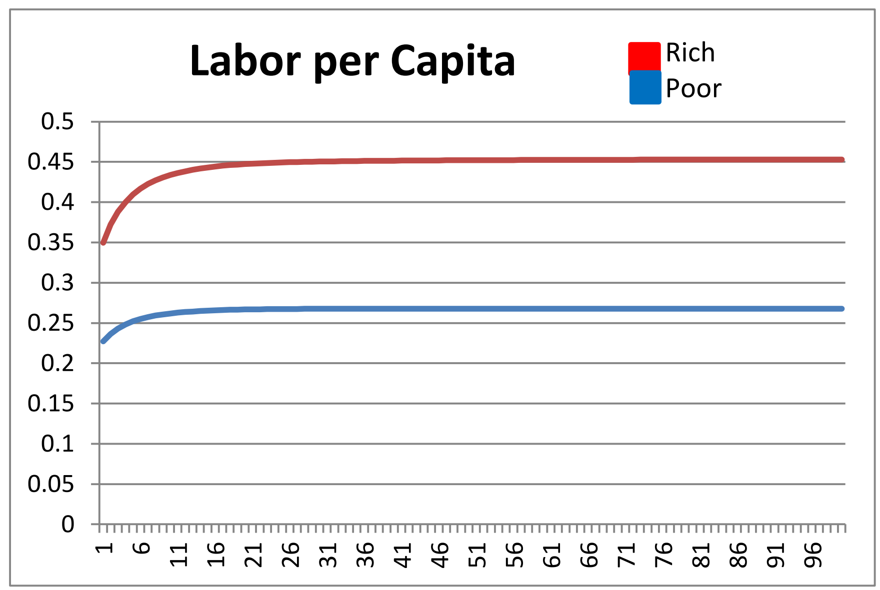


Fig. 13. Output per Capita with Inequality

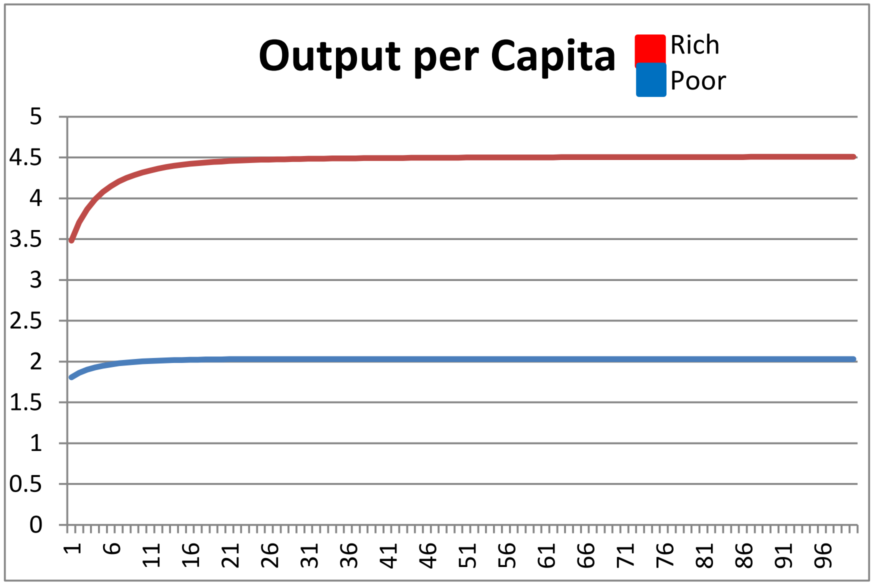


Fig. 15. Capital Productivity = Output/Stock with Inequality

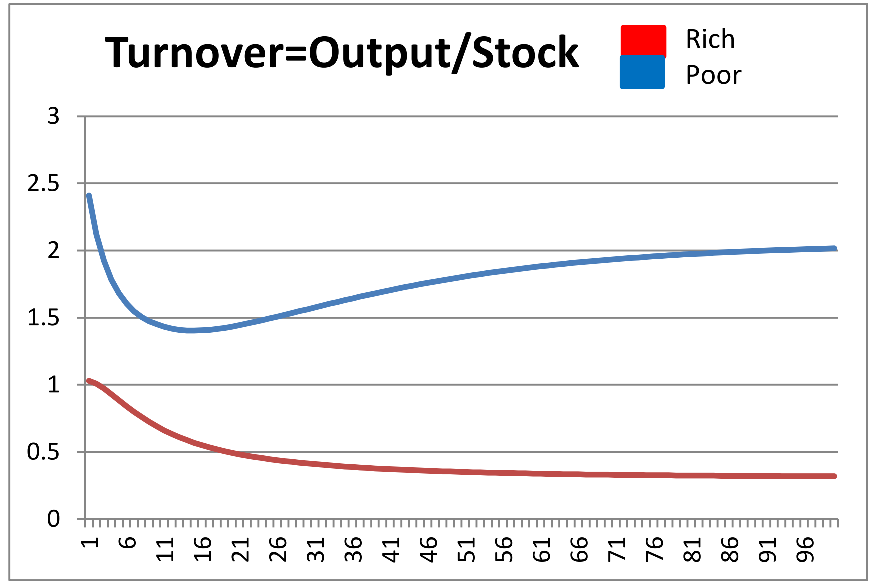


Fig. 14. Labor Productivity = Output per Manhour with Inequality

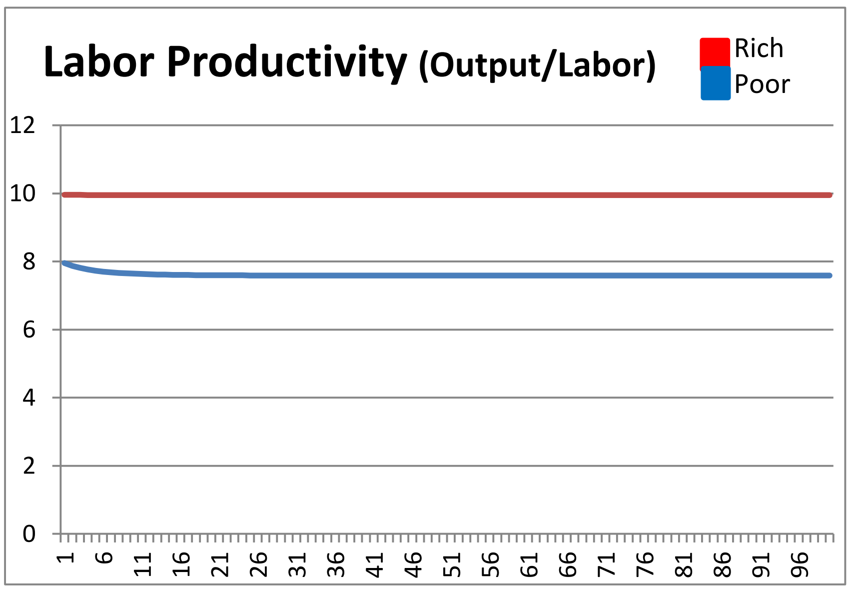


Fig. 16. Land Productivity = Output per Acre with Inequality

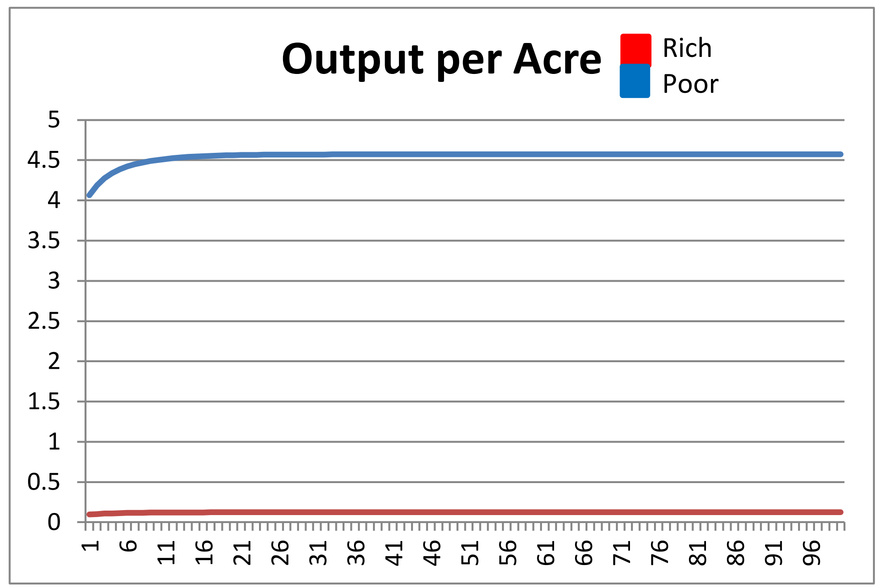


Fig. 17. Land Value with Inequality

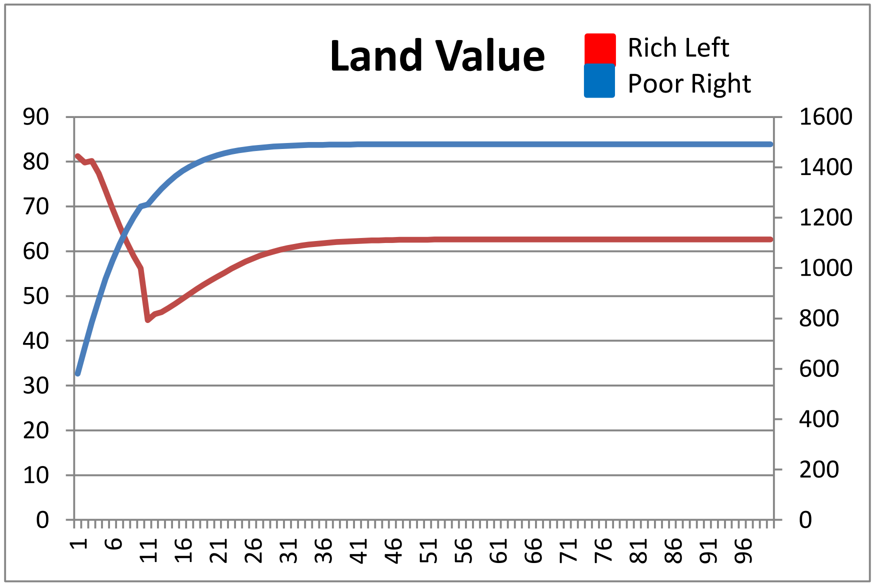


Fig. 18. Wealth = Land Value + Capital Stock with Inequality

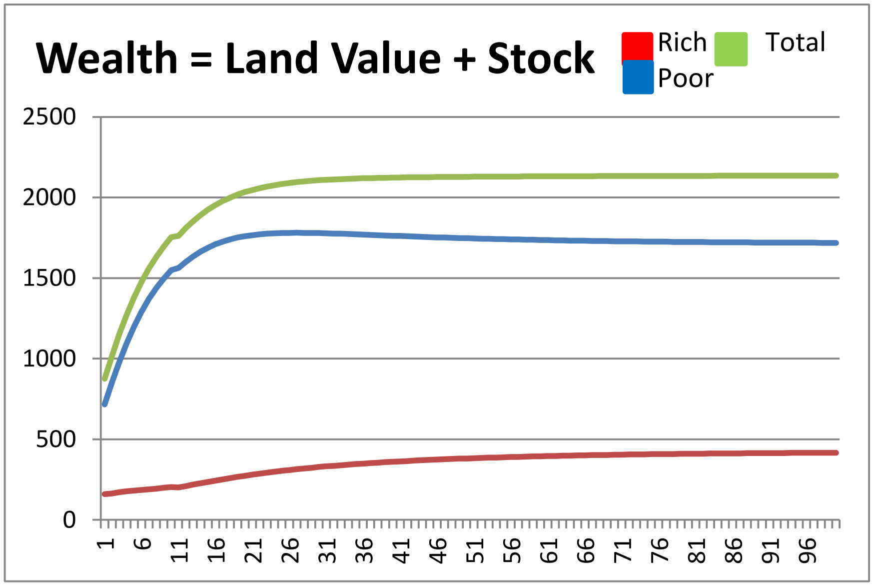


Fig. 19, Output with Shock, with Equality

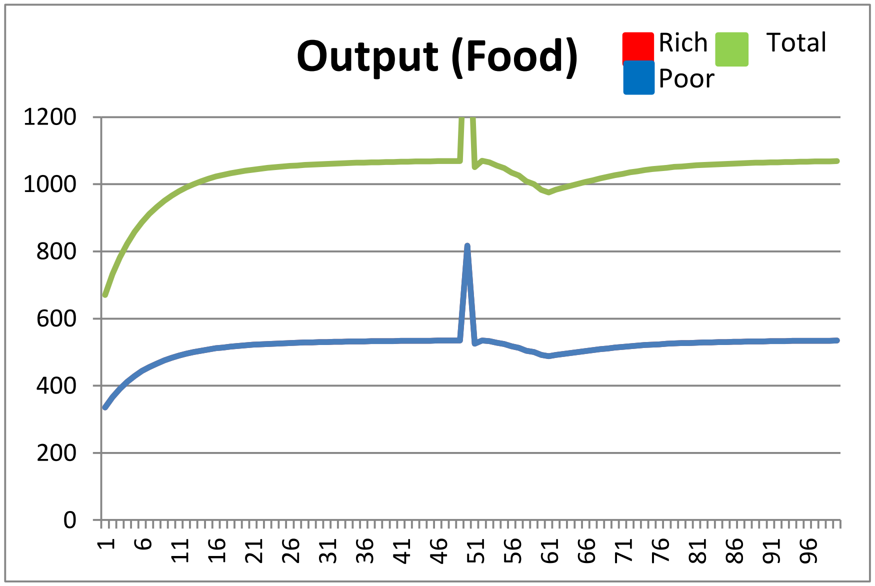


Fig. 21, Output with Shock, with Inequality

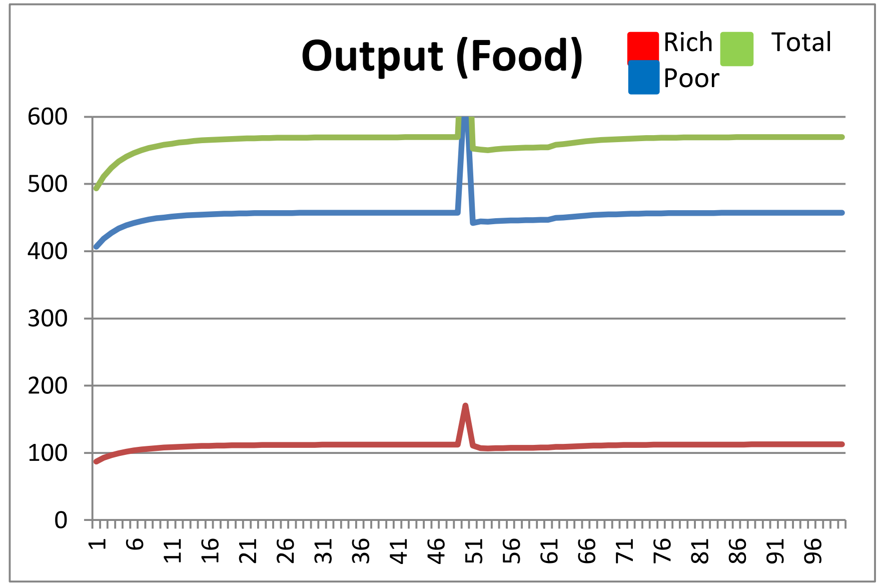


Fig. 20, Capital Stock with Shock, with Equality

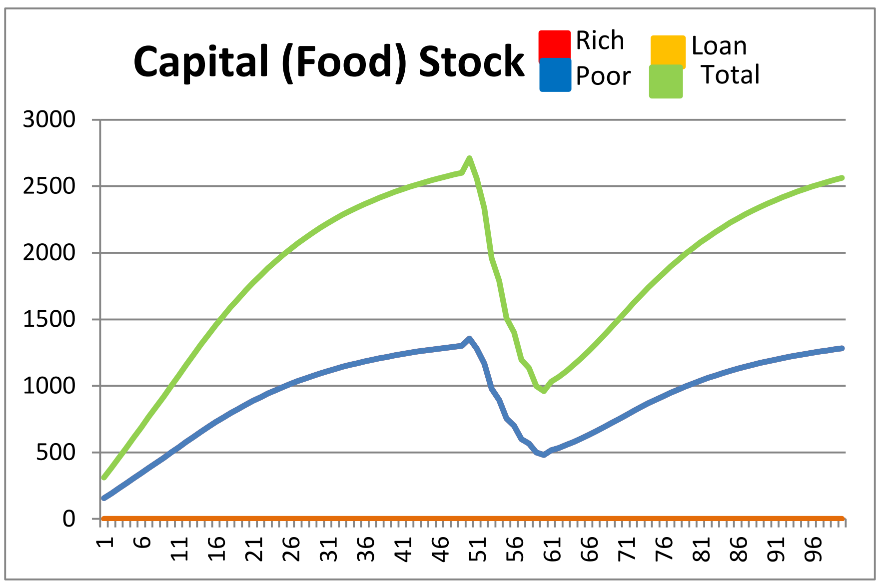


Fig. 22, Capital Stock with Shock, with Inequality

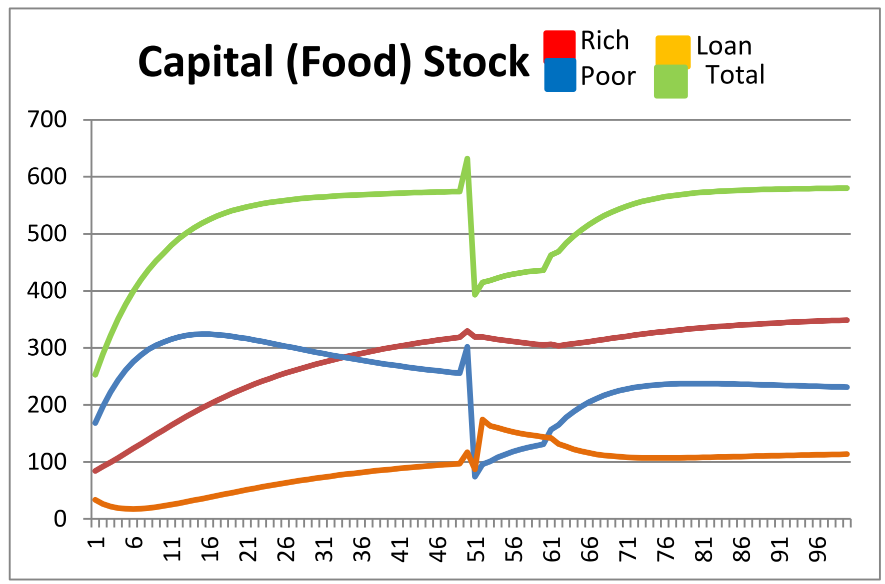


Fig. 23, Wage Rate with Shock, with Equality

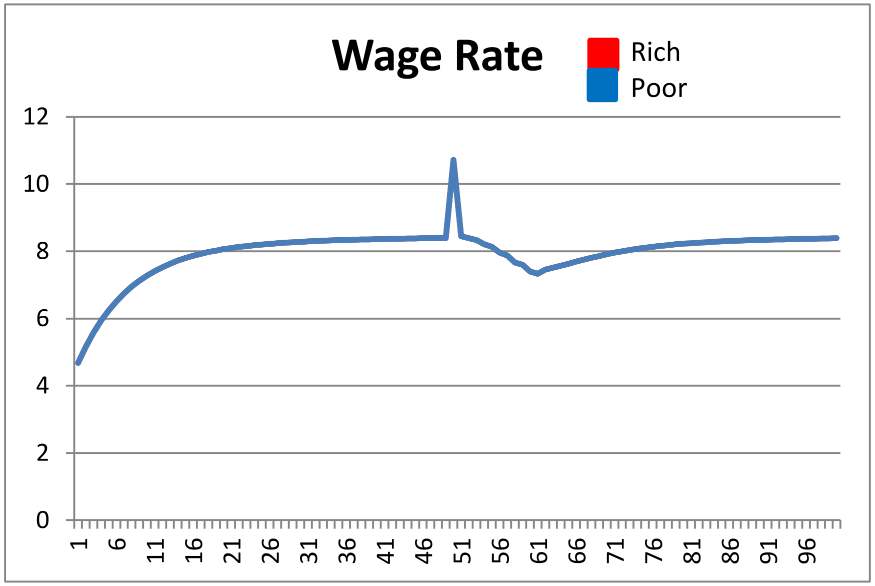


Fig. 25, Wage Rate with Shock, with Inequality

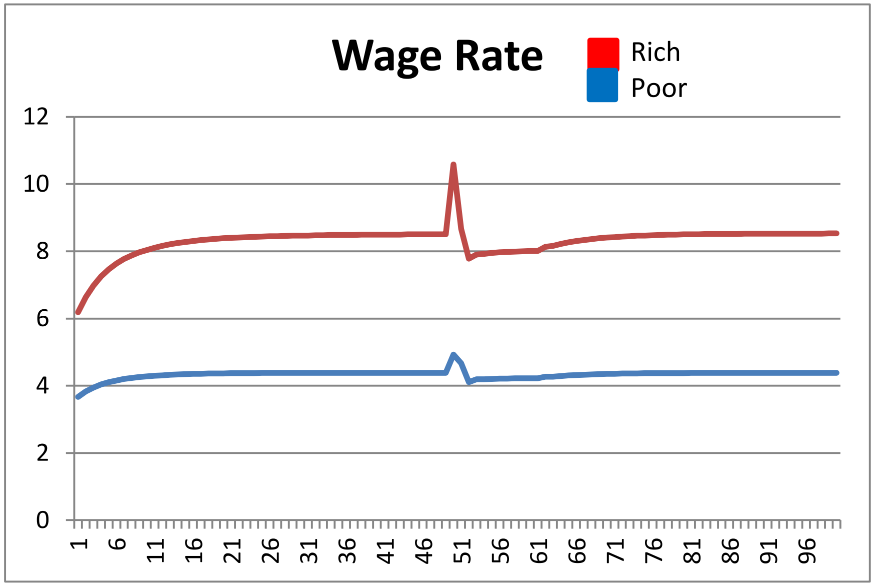


Fig. 24, Discount Rate with Shock, with Equality

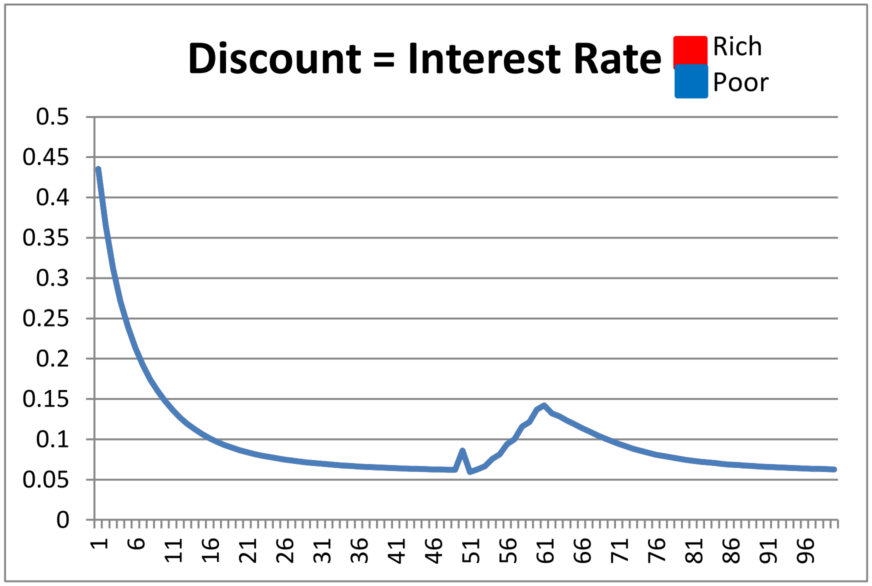


Fig. 26, Discount Rate with Shock, with Inequality

Fig. 27, Labor Supply with Shock, with Inequality

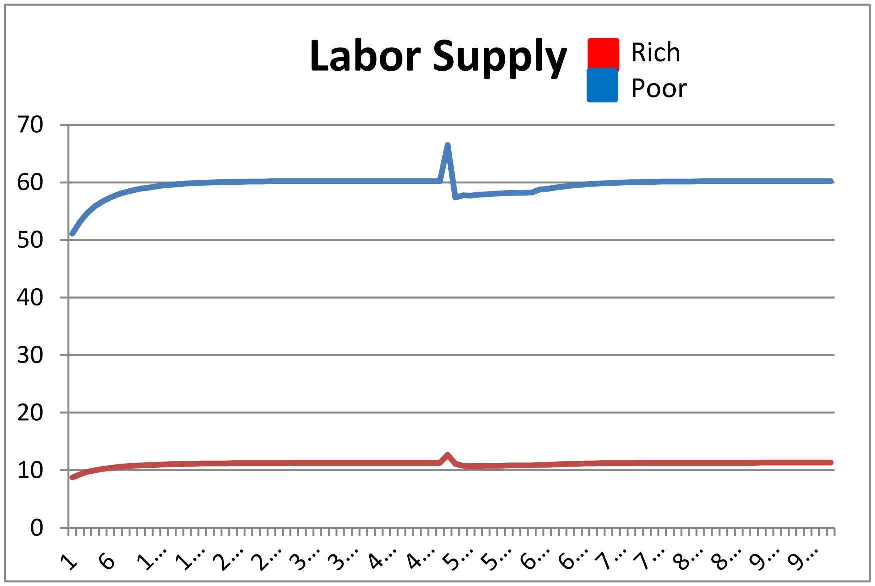


Fig. 29, Profit, Interest, and Wage Income, with Inequality

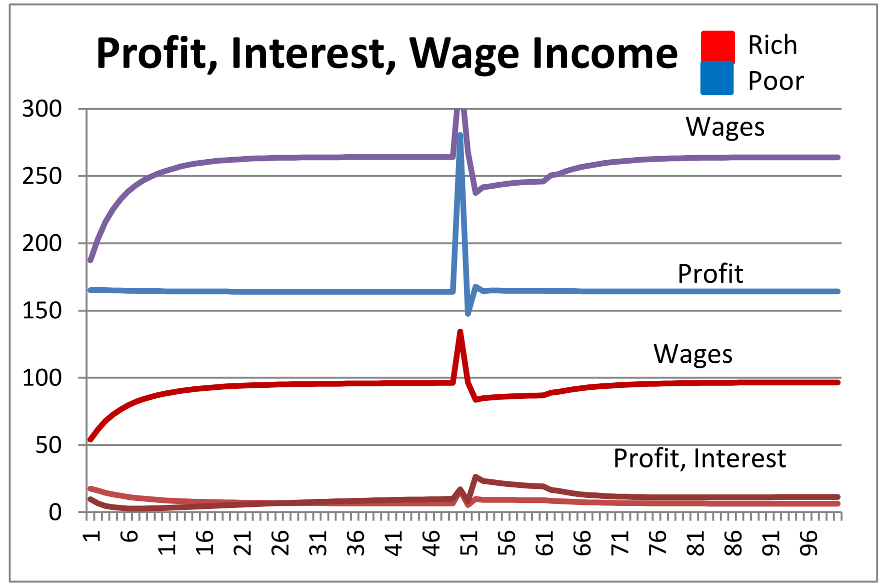


Fig. 28, Consumption with Shock, with Inequality

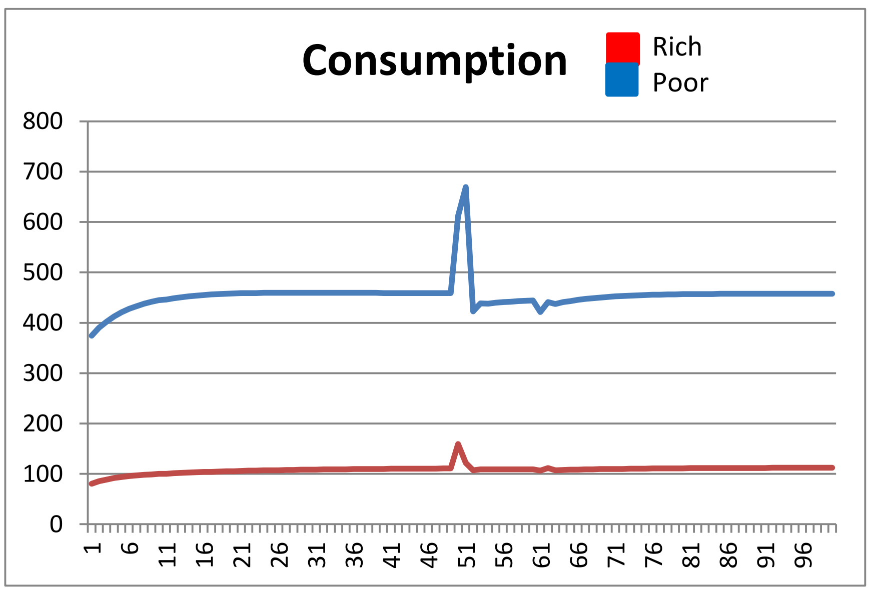


Fig. 30, Saving with Shock, with Inequality

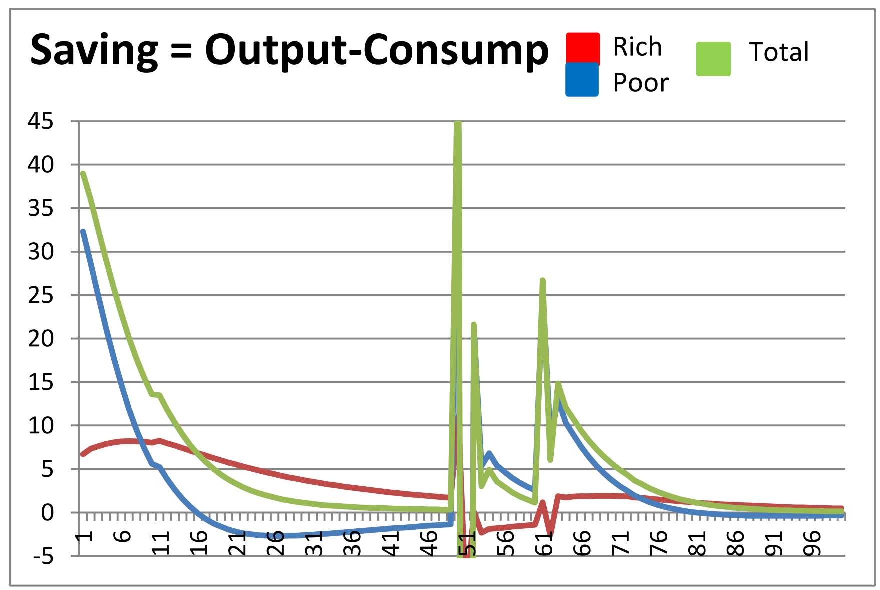


Fig. A1, Labor Supply As Function of Wage and Rent

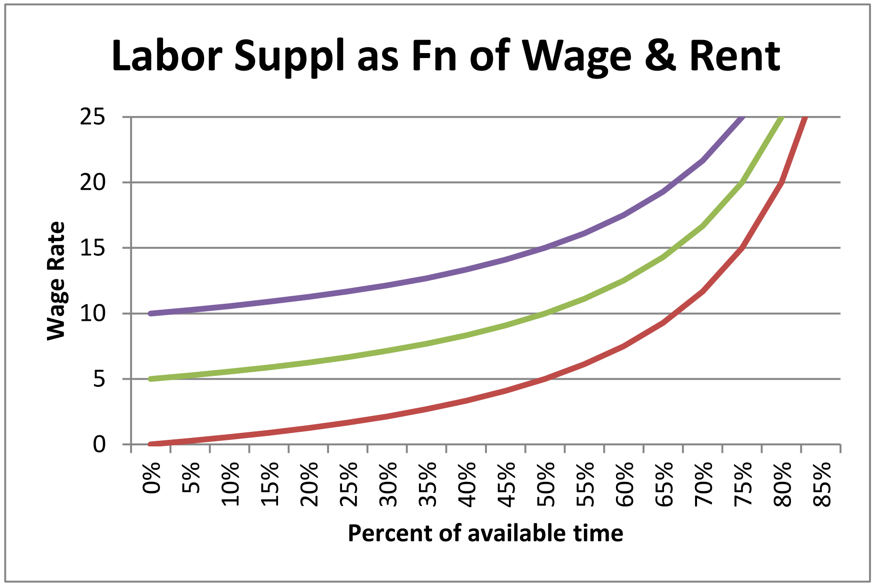
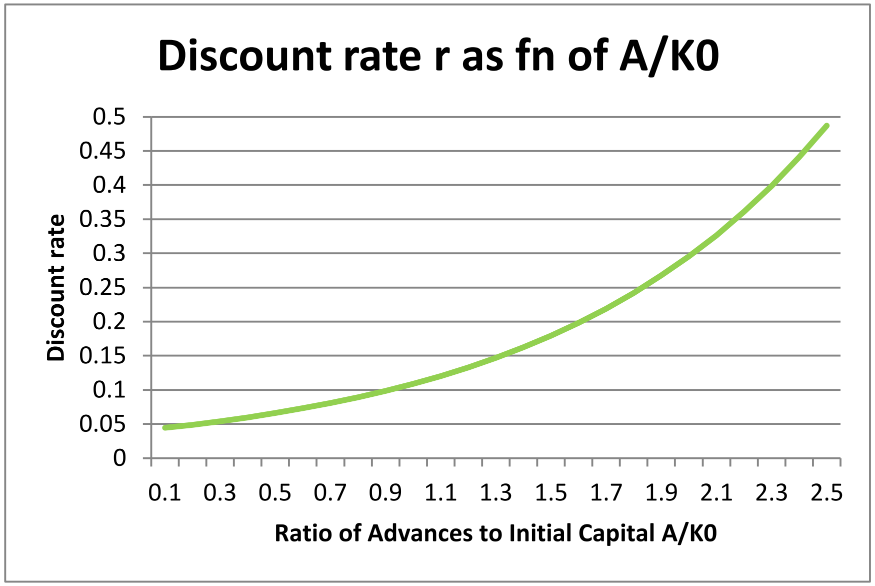


Fig. A2, Discount Rate As Function of Labor to Stock Ratio



Appendix A   2/21/2016 (refers to figures that have been renumbered)

Equation Chapter 1 Section 1

Appendix A

Imagine a uniform area of land, T, occupied by a uniform population of N individuals. Assume these individuals are simple "farmers"—they are simultaneously workers, landlords and capitalists. They generate a labor supply, L, such that $L/N < D$, the maximum possible hours of work in a day or other period of production. They receive a wage, w. As in the classical models, they produce an annual output of a single consumption good, "corn." Assume with the classical models that production takes a year, and that wages must be laid out in advance.

A.1 Production Function

Assume a well-behaved production function, depending on land, T, and labor, L:

$$Q = \varphi(T, L); \varphi_1 > 0; \varphi_2 > 0; \varphi_{12} > 0; \varphi_{11} < 0; \varphi_{22} < 0 \qquad (1.1)$$

For the simulation model, I use a simple, tractable function as follows:

$$Q = q_0 L(1 - q_1 \frac{L}{T}) \qquad (1.2)$$

where q_0 and q_1 are constants.

This function is linear homogeneous (although that is not necessary) and more plausible than a Cobb-Douglas function.

I can introduce economies of scale by adding a power α>1 to the production function, thus:

$$Q = q_0 L^{\alpha}(1 - q_1 \frac{L}{T}) \qquad (1.3)$$

The marginal product of labor is:

$$\varphi_2 = q_0(1 - 2q_0 \frac{L}{T}) \qquad (1.4)$$

Thus if the ratio of labor to land gets too high, the marginal product of labor can actually become negative—one can imagine surplus workers trampling the crops! (Cobb-Douglas would allow us to stack workers on a postage stamp while still maintaining output).

A.2 Labor Supply Function

How do the *N* farmers supply labor? It is conventional to express labor supply as a function of wage and total income, with total income including both labor income and exogenous non-labor income. There is an assumed underlying utility function in food and leisure. I will assume for simplicity that labor supply can be written as just a function of wage, *w*, exogenous income, *E*, and *D*, "day" the maximum time available in a period. I use *E* for that exogenous income, since in the model it is (economic) rent. So the labor supply equation for one individual can be written:

$$\frac{L}{D} = \gamma(w, E) < 1; \gamma_1 > 0; \gamma_2 < 0 \quad (1.5)$$

For a population of *N*, the supply is just *N* times the individual supply. The labor supply for a given period necessarily curves upwards towards the physiological limit, *D*. Assume it does not "bend backwards" as long as *E* remains constant. However, assume that the higher the non-wage income—ie profits or rents—the higher the labor supply curve. In other words, the higher non-wage income, the less labor will be supplied at a given wage. Below a certain wage, no labor will be supplied at all. Figure 1 shows labor supply as a function of wage and rent.

This upward-curving shape implies that the lower the labor supply, the greater the effect of a change in the wage. By the same logic, the lower the labor supply, the greater the effect of a change in rent. In other words, on the flat portion of the curve a small change in wage or rent has a large effect on labor supply, but little effect on the steep part; elasticity falls along a supply curve from left to right. This difference in elasticity implies that the more the "economy" operates in the flat portion of the labor supply curve, the more it will be more disrupted by fluctuations. Hence low-wage poor farmers are more susceptible to disruption than high-wage rich farmers.

I use a simple, tractable labor supply function for the simulation model. For one individual:

$$\frac{L}{D} = \frac{wD - E}{(w + w_0)D - E} < 1 \qquad (1.6)$$

--where D is the maximum time, *w* is the wage, w_0 is a constant, and *E* is rent. Obviously, if *E >= wD*, labor supply is zero. *NL/D* is the labor supply for a population of *N* laborer/farmers.

Wage *w* can also be expressed as the inverse function of *L/D* as follows:

$$w = \frac{w_0 L / D}{(1 - L / D)} + ED \quad (1.7)$$

As it should, *w* “explodes” as *L/D* approaches one. See Fig. A1.

A.3. Discount Rate

In a multi-period model, there must be a discount rate, or multiple discount rates to compare future to present values. See Fig. A2.

If discount rate, *r*, is not given, where does it come from? And how does it relate to the corn inventory at the beginning of each period?

In the classical models, profit is a residual after capital is advanced to pay rent and wages. Profit rate is profit divided by initial stock of capital, assumed to be 100% advanced. Profit rate in turn determines interest or discount rate. There is no concept of supply and demand for capital.

In modern microeconomics, interest or discount rate for the supply of capital depends on consumers’ preferences between present and future consumption, with future consumption being discounted relative to present. In macroeconomics, discount rate depends on preferences between money and bonds. Neither formulation lends itself to conveying the wages-fund concept—that a given stock of consumption goods—“advances”—must carry consumers over a certain period before it can be replenished.

I use a formulation drawn from the wages-fund:

$$r = \eta(\frac{A}{K_0}); \eta' > 0; \eta'' > 0 \qquad (1.8)$$

where r = discount rate, K_0 is initial corn (ie working capital) and *A* stands for Advances. Discount rate is thus an increasing function of the ratio of advances to initial capital. In the classical model advances equal the wage bill. Thus $A = wL$. (In Section I., which introduces credit, *A* will include loans.)

This is an unfamiliar approach. Usually we write a production function that depends on land, labor and capital (or just labor and capital). Then we set the marginal product of capital equal to the discount rate, which is given, to determine the amount of capital to be supplied. We assume that a high enough rate will call forth whatever quantity of capital may be desired. Here we do the opposite: we start with a given supply of real working capital, and say that the price of that capital depends on how fast we use it up in a given period—with catastrophe ensuing if we run out before the end of the period. However alien this interpretation of the discount rate seems to us, it is consistent with the classical model—and would of course seem perfectly natural in famine-plagued parts of the world.

A literal interpretation of the classical model suggests a discount rate function that behaves like the labor supply function, rising steeply toward an absolute limit so as to foreclose the possibility that Advances exhaust initial working capital K_0. In practice a functional form of

$$r = r_0 e^{\frac{A}{K_0}} \qquad (1.9)$$

works better in the simulation model—even though it allows $A/K_0 > 1$. This form can be rationalized by assuming that in an emergency some corn can be harvested early, or emergency stocks dug out of storage. (In the case of labor supply, there is no way to add even a second to a 24 hour day.)

Figure 2 shows this function, with r_0=.04—which is a reasonable number for the annual “natural” rate of interest.

Again, in this particular model, these are internal advances from the farmers to themselves, not an advance from one group to another, so the only implicit risk is that of running out of corn before the next harvest.

A.4 Consumption Function

In a multi-period model, as an accounting identity, ending capital stock each period equals beginning stock, plus production minus consumption. To model consumption, we need a consumption function. I have chosen a simple Keynesian-style function, in which consumption depends on wealth, W and current production, Q.

$$C = c_0 W + c_1 Q \quad (1.10)$$

c_0 and c_1 are constants. In the model, consistent with recent consumption function estimates (Ludvigson & Steindel, 1999), I will assume that the wealth coefficient c_0 = .04 and the output coefficient c_1 = .75.

Current production, Q, equals current income. No problem here.

But what is wealth? Wealth in each period must be initial capital K_0 plus the value of the other asset, land, T. In an infinite static model, with fixed r, land value $V = E/r$, rent divided by the discount rate. Assume that is the case here, as a reasonable approximation. That is, land value equals current rent divided by current discount rate. So:

$$W = K_0 + \frac{E}{r} \quad (1.11)$$

In a multi-period model, we can add in "adaptive expectations" and have wealth depend on some sort of running average of capital and rents during prior periods. Of course such short-sighted approximation can create instability.

A.5 Solving the Model for One Period

We can completely solve the model for one period. We just maximize rent.

$$Max: E = \varphi(T, L) - wL(1+r) \quad (1.12)$$

And we get the familiar result:

$$\varphi_2 = w(1+r) \quad (1.13)$$

The marginal product of labor equals the wage times (1+r). That is, the wage equals the discounted marginal product of labor.

Now, drawing on labor supply equation (1.5), and discount equation (1.8) we can solve for w, r, L, E, and Q. If we add the wealth equation (1.11), we can find consumption C, and from that, ending corn inventory:

$$K_1 = K_0 + Q - C \quad (1.14)$$

A.6 Solving the Model for Multiple Periods

We can now construct a multi-period simulation model, just by making ending capital of one period the beginning capital of the next.

Substituting for C in (1.14), there will be growth as long as:

$$K_1 - K_0 = (1-c_1)Q - c_0 W > 0 \quad (1.15)$$

By assumption the wealth coefficient c_0 is very small, .04 in the model, while (1- c_1) is large by comparison, .25 in the model. So the model economy will grow while wealth W is small, and decline if it is too large. There is an equilibrium position, where capital = K^*, at which there is no growth. Without the effect of wealth on consumption, with $c_1 < 1$, growth would continue forever.

If I run the model with a much larger wealth coefficient, say .15 instead of .04, I can get instability due to overconsumption of capital.

A.7 Comparison to a static infinite model.

Of course the whole point of a dynamic model is that discount and growth rates are determined endogenously every period. However a static infinite model, with discount and growth rates given, yields similar equations. That makes it more plausible to assume short-sighted one-period maximization in a dynamic model.

In an infinite static model, we must either assume that each future period is identical or that there is a uniform growth rate, $g < r$. (If $g >= r$, the model will blow up.) Land value is the discounted present value of future output minus labor costs:

$$V = -wL + \frac{\varphi(T,L) - wL}{1+r} + \frac{\varphi(T,L)(1+g) - wL}{(1+r)^2} + \ldots \quad (1.16)$$

Provided $g<r$, this expression converges to:

$$V = \frac{\varphi(T,L)(1+g)}{r-g} - \frac{wL(1+r)}{r} \quad (1.17)$$

Provided $g<r$, the general solution to the maximization problem is:

$$\varphi_2 = w(1+r)(1-\frac{g}{r}) \quad (1.18)$$

Absent growth, $g = 0$, infinite present value maximization yields the same equation, (1.13) as the one-period model.

$$\varphi_2 = w(1+r)$$

Notice that with $g > 0$, the marginal product of labor is lower than with $g = 0$; that is expectations of growth lead farmers to apply more labor and produce more.

A.8 A Multi-Period Two-Region Model: Endogenous Growth Only, No Lending

Given the above equations, and plausible assumptions about parameters and starting values, we can turn the computer loose, generating each period from ending values of the prior period. I used GAMS, General Algebraic Modeling System. A truncated version, adequate for small models, can be downloaded free from the GAMS website, www.gams.com.

Imagine an economy with two completely independent regions, or alternatively, a rich and a poor nation side by side. The first "rich" region, has land 900, and population 25. The second "poor" region has land 100 and population 225. So 10% of the population has 90% of the land. Run the model over 100 periods. Assume all growth is endogenous, resulting from the gradual accumulation of corn stocks, beginning at an arbitrary low starting point in both regions.

As shown in Figure 3, in both Richland and Poorland, output rises rapidly at first, then flattens off. Total output is much greater in Poorland; with 90% of the population but only 10% of the land, Poorland accounts for about 80% of output. However output per capita is greater in Richland (about 5 units per capita versus 2 units per capita). As shown in Figure 4, capital continues to accumulate, but at a decreasing rate, tapering off faster in Poorland. Figure 5, shows labor supply. Total labor supply is much greater for Poorland than for the rich. However, individually the rich work much harder, (about 50% of the time, as opposed to 30% for the poor). This isn't surprising, since, as shown in Figure 6, wages of Richland are more than double those of Poorland—due to the higher marginal product of labor.

Figure 7 shows discount rates for the rich and poor regions. Discount rates fall in both regions as capital accumulates. However, the rate in Richland is everywhere lower than that in Poorland—even near the beginning of the run, where Poorland has accumulated more capital than Richland.

A.9 A Multi-Period Two-Region Model: Endogenous Growth Only, With Lending

Now assume Richland can lend capital (corn) to Poorland. There are two cases: In the first case, assume there are no transactions costs to impede lending so that discount rates of the rich and poor region are identical. That is, referring back to equation (1.8):

$$r^P = \eta\left(\frac{w^P L^P + Z}{K_0^P}\right) = r^R = \eta\left(\frac{w^R L^R - Z}{K_0^R}\right) \qquad (1.19)$$

where Z is the quantity of corn loaned by Richland to Poorland at the beginning of each period, and returned with interest at the end of the period. (Superscripts P and R refer to Poor and Rich.). Figure 8 shows how the interest rates have converged to a rate between the separate rates in Figure 7.

Figure 9 however, shows a remarkable contrast with Figure 4 above. In Poorland, capital accumulation rises at first more rapidly than in Richland. But instead of leveling off at a high point, as it does in Figure 4, capital accumulation actually falls again, before eventually leveling off. The inhabitants of both regions are in fact better off due to lending, as they should be. Output and consumption are both up—though not enough to show up clearly on a graph. A reasonable interpretation of the decline in poor region capital accumulation shown in Figure 9 is that with relatively cheap capital now available from Richland, Poorland no longer needs so much accumulation. (This result is consistent with findings of Banerjee and Duflo, that very poor people remain chronically in debt to moneylenders at very higher interest rates, when they could "easily" reduce that debt by cutting consumption of items like tea, tobacco and beer, or by forgoing parties with friends (Banerjee & Duflo, 2011)).

In the second case, assume that lending is impeded, as it is in the real world, by transactions costs, moral hazard and adverse selection. Basically, the larger a loan in proportion to the borrower's collateral, the greater the risk of default. Also, the higher the interest rate, the more likely the borrower is a bad credit risk, without better options elsewhere. Such barriers to lending are modeled in great detail by Stiglitz and Greenwald (Stiglitz & Greenwald, 2003). So lending does not go as far as it would without such barriers. To keep things simple, I have modeled only the increase in possible loss proportional to borrower's collateral. Assume a loss function:

$$\chi(\frac{Z}{W^P}); \chi \le 1, Z = 0 \Rightarrow \chi = 1, \chi' < 0 \quad (1.20)$$

Equation (1.12) for Richland now becomes:

$$Max: E^R = \varphi(T^R, L^R) - w^R L^R (1+r^R) + Z(1+r^P)\chi(\frac{Z}{W^P}) - Z(1+r^R) \quad (1.21)$$

Maximizing with respect to Z yields:

$$0 = (1+r^P)\chi - (1+r^R) + Z(1+r^P)\chi' \quad (1.22)$$

I used a very simple function for the simulation, where *pz* is a constant:

$$\chi(\frac{Z}{W^P}) = (1 + pz\frac{Z}{W^P})^{-1} \qquad (1.23)$$

So if loan amount $Z>0$, a gap emerges between the discount rates in Richland and Poorland. Figures 10 and 11 show the consequences for discount rates and capital accumulation. The gap between discount rates is smaller than in Figure 7. Capital accumulation falls slightly in Poorland, but not as much as in Figure 9, with no barriers to lending.

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