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Unequal Distribution as a Cause of Market Failure--Are Land Taxes the Remedy?

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Given transactions costs and economies of scale in production, inequality of land ownership necessarily creates a labor and land market failure; the greater the inequality, the more severe the failure. This failure explains the regressive land use characteristic of less-developed agriculture: the larger a landowner, the lower the ratio of labor to land, and the less the output per unit area for constant land quality. A land tax, far from the neutral tax of conventional theory, imposes income and marginal effects that counteract this market failure. An output tax compounds market failure.

In his 1879 classic *Progress and Poverty*, Henry George proposed that land taxes should replace all other taxes. Many economists before and after Henry George have favored land taxes, sometimes equating them to the ideal lump sum tax. Development economists often recommend land taxation as a less drastic means than land reform to bring underused land into production [de Janvry, 1993]. However, in a recent paper from a World Bank symposium, “Land Taxes, Output Taxes, and Sharecropping: Was Henry George Right?” Karla Hoff challenges that conventional wisdom [Hoff, 1993]. She constructs a model to show that, given insurance market failure, a land tax plus a small output tax may be Pareto-superior to a pure land tax, because an output tax permits risk sharing. Therefore, she concludes, “Henry George was wrong!”

Hoff’s model assumes equal distribution of land ownership; Henry George and other land tax supporters concern themselves with highly *unequal* distribution. But Hoff raises an important question: “*What is the impact of land taxation in the presence of market failure?*”

Henry George focused attention on a phenomenon already clearly identified by Adam Smith: *regressive land use*. That is, holding land quality constant, larger property owners use their land less intensively than do smaller ones. Smith observed that:

> To improve land with profit, like all other commercial projects, requires an exact attention to small savings and small gains, of which a man born to great fortune, even though naturally frugal, is very seldom capable... He embellishes perhaps four or five hundred acres in the neighborhood of his house, at ten times the expense which the land is worth after all his improvements; and finds that if he was to improve his whole estate in the

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> same manner, and he has little taste for any other, he would be a bankrupt before he has finished the tenth part of it...
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> A small proprietor, however, who knows every part of his little territory, who views it with all the affection which property, especially small property, naturally inspires, and who upon that account takes pleasure not only in cultivating but in adorning it, is generally of all improvers the most industrious, the most intelligent, and the most successful. [Smith, 1952]

George described a pattern of underuse or nonuse of large land-holdings around the world, from the newly-opened farmlands of California, to the estates of absentee British aristocrats in Ireland, to the feudal holdings of the *zamindars* of India--who served as tax-collectors for the British East India Company. George saw more than mere inefficiency: he saw the root cause of the poverty that persisted and even increased in the face of growing economic prosperity. He emphatically refuted the Malthusian doctrine that attributed the starvation of Irish or Hindu peasants to “overpopulation,” pointing out that during the Irish potato famine, Ireland remained a major exporter of wheat. In his view, landlords withheld land from use, forcing down wages to subsistence. A land tax, together with an elimination of other taxes, would force landowners to release idle land for productive use, eliminating poverty.

Progress and Poverty proved immensely popular; millions of copies were sold, translated into all the major languages. George toured the world, speaking before huge crowds. His followers successfully instituted land taxes in parts of California and Pennsylvania, Australia, South Africa and Denmark. His ideas inspired political reformers well into the 20th century. George himself died suddenly in 1895, in the middle of a probably successful campaign for Mayor of New York City.

Despite his popularity and influence, George received little recognition as an economist. His attack on the Malthusian doctrine and the policies of the British Empire thoroughly antagonized the British establishment, including such economists as Alfred Marshall, who once publicly debated him. And a key part of his argument did not hold up very well: George explained land withholding as due to “land monopoly” or “land speculation.” As Marshall asked, how could there be a land monopoly when there were many landowners and no apparent collusion between them? [Marshall, 1883] Absentee speculators obviously bought up large tracts of land and held them unused in newly-opened California irrigation districts where land values were rapidly rising, as George noted. But unused or underused tracts could be found where land values were stable as well. Poverty persisted and inequality increased where there was economic growth, as on the American frontier a hundred years ago. But poverty persisted in the absence of growth in the Old World or China or India.

Today, regressive land use remains a conspicuous feature of less-developed agriculture (and a still-significant feature of developed agriculture.) Data from a major survey of Latin American land ownership dramatizes the contrast between large and small land holdings. (See Table I.) The smallest holdings employ 46.5 workers per 100 hectares, vs. 1.43 per 100 hectares on the largest, about *35 times* as many! 55% of the smallest land holdings are cultivated, but only 16% of the largest. Yet the larger holdings occupy better land, more suited to crops than to livestock. In these countries over half the food production comes from smaller holdings (mini and family farms), which occupy less than a fourth the land. [Feder, 1971].

Table I
Distribution of Land and Labor in Seven South American Countries 195060.

Farms	Farmland 489.5 Million Hectares	Workers/100 Hectares	% of Land Cultivated
Mini Farms	2.3%	46.50	55%
Family Farms	20.8%	5.19	29%
Smaller Estates	24.1%	5.08	33%
Larger Estates	52.7%	1.43	16%

There have been many efforts to correct underuse of large holdings by land reform: redistributing land in small parcels. The two most thorough land reforms occurred in Japan just after World War II and in Taiwan in the early '50's, both--by no coincidence--under occupying armies. More limited land reforms, as in South Korea, Mexico, Bolivia, and Egypt have also yielded good results.[Dorner, 1971] But most land reform attempts have foundered on the political power of large landowners. [de Janvry & Sadoulet, 1993].

Meanwhile, economic theory has caught up with the problem. The last twenty years in particular have seen a rapid development in theories of transactions costs, property rights, principal-agent interactions, market failure, missing markets and so forth. These now make it possible to explain regressive land use with conventional neo-classical methods. They also make it possible to predict the consequences of imposing land taxes and removing other taxes.

Capital market failure has long offered a partial explanation for regressive land use. As Rainer Schickele observed many years ago, when banks make loans, “The principle of allocation is collateral security, not marginal productivity...These two principles tend to work at cross purposes: with increasing collateral security, the marginal productivity of capital tends to decline, and vice versa. Instead of allocating capital to where it is scarce, our credit system allocates it to places where it is ample.” [Schickele, 1943, p 240]. Mason Gaffney has extensively investigated the effects of capital market failure on land use. [Gaffney, 1956, 1961, 1975]

But capital market failure cannot in isolation explain regressive land use. It does not permit a general equilibrium; there must also be a labor market failure to make the capital market failure effective. That is, suppose capital market failure prevents poor individuals from buying or renting land from the local landlords. These individuals should be able to work for the landlords at a wage and intensity such that the capital market constraint does not bind, and productivity remains unaffected.

Enter supervision costs, now very familiar from the principal-agent literature. An agent has an incentive to shirk. In an uncertain environment, necessarily the case in agriculture, the principal cannot verify the agent's performance without monitoring him. Supervision costs create a labor market failure that balances capital market failure. In fact, capital market failure itself ultimately arises from supervision costs in the capital markets.

The first published neo-classical model that clearly embodies the concept of labor market failure balancing capital market failure is that of Mukesh Eswaran and Ashok Kotwal [Eswaran and Kotwal, 1986]. They develop a model of five agrarian classes depending on working capital, with class boundaries determined by Kuhn-Tucker conditions: pure laborer, laborer-cultivator, self-cultivator, small capitalist and large capitalist. The laborer-cultivator cultivates a small plot and hires labor out, the self-cultivator neither hires in nor out. The small capitalist hires in, works his own land, and supervises his employees. The large capitalist hires in and only supervises his employees. Working capital is partly given, and partly borrowed in proportion to land-ownership, which is exogenous. Eswaran and Kotwal's model clearly predicts regressive land use.

Eswaran and Kotwal in turn acknowledge a debt to John Roemer, who in 1982 proposed a similar scheme of five classes dependent on capital ownership. [Roemer, 1982], as well as to Pranab Bardahn, who immediately applied Roemer's scheme to Indian peasant society [Bardahn, 1982]. Roemer's models are rather complex linear programming exercises involving only labor and capital. Roemer dismisses land altogether as a factor of production, by assuming an unlimited supply. This perhaps explains why Eswaran and Kotwal downplay land to focus on working capital, even in their title-- "Access to Capital and Agrarian Production Organization" instead of "Access to Land.." In fact they conclude that "the creation of institutions capable of accepting as collateral future crops rather than owned land-holdings would prove to be an effective tool for removing poverty as well as for improving efficiency." [p 196].

Others, notably Michael Carter, have elaborated on Eswaran and Kotwal's model. [Carter and Kalfayan, 1989; Carter and Mesbah, 1993; Wydick, 1994].

Working independently, I constructed models to explain regressive land use as a consequence of barriers to trade between individuals with different wealth endowments. [Cleveland, 1984]. To illustrate the basic concept, imagine a collection of Robinson Crusoes each occupying his own island an hour or two's canoe paddle from the others. If the islands vary in size, the occupants of larger islands may hire those of smaller islands. But the physical barrier of the canoe trip obviously causes regressive land use. Moreover, the occupants of the smaller islands necessarily experience a lower marginal product of labor and hence lower effective wage. In a simple agricultural model, wealth endowments likewise consist entirely of land; capital is unnecessary. The barriers to trade consist of supervision costs required when one person's labor combines with another person's land. The time and energy barrier of supervision costs mean that as land size increases, intensity of land use falls and the owner's wage rises.

I now return to these simple agricultural models to explore the question raised by Hoff: "*What is the impact of land taxation in* the *presence of market failure?*"

I shall proceed as follows:

Part I develops a simple model of a consumer-laborer, in one period, and a consumer-laborer-investor, in many periods. The consumer-laborer consumes food and supplies labor, subject to the constraint that there is only so much time in a day. In the one period model, the consumer-laborer works for an exogenous wage, and receives an exogenous lump sum profit. In the many-period model, he works in each period for a wage, receives a profit, and can trade income between periods at exogenous interest rates.

Part II combines the consumer-laborer with land to create a “farmer.” The farmer produces food from his land, according to a production function of land and labor. This production function shows economies of scale, especially at small land size. The farmer can hire labor out or in, and rent land in or out. However, transaction costs keep economies of scale from blowing up this little world: Hired-in labor is less effective that the farmer’s own labor, because it must be supervised. Rented-in land cannot be obtained at a fixed rate: rather, the rate increases with the quantity rented and with the ratio of the quantity rented to quantity owned--so that the smallest farmers pay the highest rents. This rental relationship occurs simply because farmers who rent out must supervise their tenants.

In a single period model, these assumptions produce the familiar pattern: Assuming uniform quality land, the larger the farmer the higher the average and marginal product of labor, and the lower the average and marginal product of land. Land belonging to large farmers appears “underused” compared to that of small farmers; small farmers receive a lower effective wage for their labor.

In a multi-period model, economies of scale and transactions costs keep the one-period pattern stable. After all, a large farmer, with his lower marginal product of land, should be able to sell a bit of land to a small farmer, to their mutual advantage, circumventing the transactions costs inherent in hiring or rental. But economies of scale enforce a *minimum parcel size*; any parcel, for example, must permit access and allow room for agricultural implements to be used effectively. Then, the costs of selling land contain a substantial fixed component that also enforces a minimum parcel size; boundaries must be surveyed, title defined and transferred, price haggled over, brokers and (often) taxes paid. The minimum parcel size requirement may effectively preclude small farmers from purchasing additional land (or landless workers from purchasing any land). The minimum-parcel-size barrier also raises small farmers’ effective internal discount rate compared to that of large farmers.

Thus transactions costs and economies of scale create a land and labor market failure in proportion to inequality in land ownership. Regressive land use mirrors a regressive internal discount rate and a progressive internal wage, that is, the more land an individual owns, the lower his discount rate and the higher his wage.

Unlike the model of Eswaran and Kotwal, these models depend on land and labor only, because production is instantaneous in each period. In the multi-period model, land of course has a *capital value*--the discounted present value of its future marginal product. But that does not make land into capital, any more than a ball-player’s contract turns his labor into capital. (As Mason Gaffney points out, economists all too commonly muddle land together with capital. [Gaffney, 1994]).

Part III contrasts the impact of output and land taxes on the farmers of Part II. The two taxes do in fact have largely opposite impacts. The output tax predictably discourages effort, lowering labor supply and thus output. With reduced labor, the marginal product of labor rises. The land tax, by contrast, increases labor supply and output per acre, while lowering the marginal product of labor. Thus, given market failure, the land tax is no longer neutral, but has positive marginal effects!

In a many period model, output and land taxes also have opposite impacts. An output taxes raises the transaction cost barrier that prevents small farmers from purchasing a minimum size parcel of land. A land tax lowers that barrier. In fact a land tax accomplishes directly what Eswaran and Kotwal propose to do indirectly by reforming credit markets: it makes the land accessible to a farmer depend more on future crops than on present land-holdings.

Part IV lays out a numerical general equilibrium model for a simple economy consisting of 100 identical farmers occupying a uniform area of land. The individuals are divided into five groups, consisting of 5, 10, 15, 20, and 50 farmers respectively. The groups' share of land varies according to a formula from complete equality to the top 5% having almost 60% of the land. A required minimum parcel size renders an increasing fraction of the bottom 50% "landless" as distribution becomes more unequal. Farmers who hire in labor face a supervision cost in that the "effectiveness" of hired labor is reduced by a factor proportional to the ratio of own labor to hired labor. An optimization program (GAMS) computes the equilibrium over the range of distributions for three cases: no taxes, a 50% output tax, and a land tax (set to equal the output tax at equal distribution.) It is assumed that the taxes simply leave the economy.

As predicted, as the top 5%'s land share increases, so does their wage, their income, their output, their profit and *the hours that they work*. However, their ratio of labor to land decreases, as does their output per acre, their marginal product of land, and the effectiveness of their hired labor. For the entire economy, greater inequality brings lower effective labor supply, lower income, lower output, and lower profit.

The 50% output tax only slightly intensifies the relative effects of unequal distribution of land. However it proportionally reduces everything by about 50%, including the marginal product of land. The land tax, by contrast, sharply increases output even at the extreme of inequality. It reduces the income disparity caused by unequal land ownership. It increases labor and the marginal product of land. Most dramatically, at a relatively small degree of inequality, the land tax makes the farms of the top 5% unprofitable, as the marginal product of their land falls below the land tax rate. Thus if transaction cost barriers to sale maintain unequal distribution of land, an output tax reduces the benefit of overcoming the barriers; a land tax increases the benefit.

Because of economies of scale in production, labor plus land shares exceed output. This apparent impossibility in fact creates no problems at all. Labor share consists of own labor share plus hired labor share; only the hired labor share is actually paid from one individual to another. Due to supervision costs, land plus hired labor share cannot exceed output and there are no riots in the streets.

Part V sketches generalizations of these models to such conditions as variable land quality. It also addresses the issue of risk raised by Hoff.

I. The Consumer-Laborer

A. The Single-Period Model

The consumer-laborer consumes food and leisure. His labor supply equals the maximum time available in a period minus leisure.

Notation 1--Consumer-Laborer
$w \ldots$ Consumer - laborer's wage
$F \ldots$ "Food," assumed to have unit price
$P \ldots$ Profit of Consumer - Laborer (exogenous)
$D \ldots$ Maximum time available for labor in a period, eg 24 hours in a Day.
$Z \ldots$ Leisure of Consumer - Laborer
$L = D - Z \ldots$ Labor supply of Consumer - Laborer
$\mathrm{u}(F, Z) = \mathrm{u}(F, D - L) \ldots$ Utility function in food and leisure

The consumer-laborer maximizes utility:

(1) $$\text{Max: } \mathrm{u}(F, D\text{-}L) \quad \text{st} \quad F = P + wL$$

First-order conditions:

(2) $$\left[\frac{\mathrm{u}_Z}{\mathrm{u}_F} - w\right] \geq 0 \quad \left[\frac{\mathrm{u}_Z}{\mathrm{u}_F} - w\right]\left[D - Z\right] = 0$$

Income of Consumer-Laborer in one period:

(3) $$y = P + wD$$

Labor can be expressed as a function of income and wage, or profit and wage, where profit here is exogenous. Assume the labor supply function approaches a limit -- *D*-- as wage increases, holding income constant, or as income decreases, holding wage constant:

(4) $$L = \mathrm{a}(y, w) = \mathrm{a}(P + wD, w) \ \ldots \ \mathrm{a}_y < 0; \mathrm{a}_w > 0; \mathrm{a}_{yw} > 0; \mathrm{a}_{yy} < 0; \mathrm{a}_{ww} < 0$$

Assume further:

(5) $$\frac{\partial L}{\partial w} = \frac{\partial}{\partial w}\mathrm{a}(P + wD, w) = \left[\mathrm{a}_y\, D + \mathrm{a}_w\right] > 0$$

That is, holding profit constant, labor supply does not bend backward. In general, assume that wage terms, a_w, dominate income terms, a_y, since any results that hold without backward-bending hold *a fortiori* with it.

B. The Multi-Period Model

Notation 2--Consumer-Laborer-Saver
$0,1,2,\ldots i,\ldots$ Periods. 0 is current period.
$C_i\ldots$ Consumption in each period.
$S_i\ldots$ Saving in each period.
$W_i\ldots$ Wealth in each period.
$\delta_1, \delta_2, \ldots \delta_i \ldots$ Discount rate between period 0 and 1, 1 and 2, etc.

The multi-period consumer-laborer utility function is:

(6) $$U = \mathrm{u}(F_0, Z_0, F_1, Z_1, \ldots F_i, Z_i, \ldots), \quad \text{where}$$
$$Z_i = D - L_i; \forall i$$

Income in each period is *P* (exogenous) profit plus total time, *D*, valued at the (exogenous) wage:

(7) $$y_i = P_i + w_i D$$

Consumption equals food, *F*, (with food price = 1), plus the value of leisure, *Z*. Consumption equals income in a one-period model. With many periods, there may be saving or dissaving in each period:

(8) $$C_i = F_i + w_i Z_i = y_i - S_i$$

The consumer-laborer is now subject to a wealth constraint, where W_0 is (exogenous) wealth and δ_i are (exogenous) discount rates:

(9) $$W_0 = C_0 + \frac{C_1}{1+\delta_1} + \frac{C_2}{[1+\delta_1][1+\delta_2]} + ... + \frac{C_i}{\prod_1^i [1+\delta_i]} + ...$$

And saving in any period equals the discounted value of the difference between this and next period wealth:

(10) $$S_i = \frac{W_{i+1} - W_i}{1+\delta_{1+i}} = W_i \frac{\delta_{1+i}}{1+\delta_{1+i}} - C_i$$

So that income is discounted return on next period wealth:

(11) $$y_i = P_i + w_i D = W_i \frac{\delta_{1+i}}{1+\delta_{1+i}}$$

Maximization yields first order conditions:

(12) $$\left[\frac{\mathrm{u}_Z^i}{\mathrm{u}_F^i} - w_i\right] \geq 0 \quad \left[\frac{\mathrm{u}_Z^i}{\mathrm{u}_F^i} - w_i\right][D - Z_i] = 0$$

(13) $$\left[\frac{\mathrm{u}_F^i}{\mathrm{u}_F^{i+1}} - [1+\delta_{i+1}]\right] = 0 \quad \left[\frac{\mathrm{u}_Z^i}{\mathrm{u}_Z^{i+1}} \frac{w_{i+1}}{w_i} - [1+\delta_{i+1}]\right] = 0$$

The first pair of equations yields a labor supply function that depends on wealth as well as current income and wage, and implicitly on all future parameters:

(14) $$L_i = \mathrm{a}(y_i, w_i, W_i)$$

The second set of equations yields a marginal rate of substitution function that depends on present consumption and future wealth, and implicitly on all future parameters:

(15) $$\mathrm{m}(C_i, W_{i+1}) = 1 + \delta_{i+1}$$
$$\mathrm{m}_C < 0; \quad \mathrm{m}_W > 0$$

Assuming homothetic time preferences, the marginal rate of substitution function will not change along a ray from the center, that is:

$$\mathrm{m}_C + \frac{W_{i+1}}{C_i}\mathrm{m}_W = 0 \qquad (16)$$

II. The Farmer

A. The Single-Period Consumer-Laborer as a Farmer

Combine a consumer-laborer with a piece of land to which he applies his labor, making him a farmer. How does farm size affect the farmer's behavior?

Notation 3--The production function:
$T\ldots$ Size of a parcel of land
$A\ldots$ Labor Applied to land parcel
$F = \mathrm{f}(T, A)\ldots$ Food output from $T\ldots \mathrm{f}_T > 0; \mathrm{f}_A > 0; \mathrm{f}_{TA} > 0; \mathrm{f}_{TT} < 0; \mathrm{f}_{AA} < 0$
$\mathrm{f}_{TT}\cdot \mathrm{f}_{AA} - \left[\mathrm{f}_{TA}\right]^2 < 0$ and $\mathrm{f} - \mathrm{f}_T \cdot T - \mathrm{f}_A \cdot A < 0$ for small T, ie, $\exists$ economies of scale.

1. Farmer Can Hire In or Out but Not Rent Land

Notation 4--Hiring **Assumptions for Farmers who Hire In or Out**
v ... Market wage $\vec{H}$ ... Hired-out labor $\overleftarrow{H}$ ... Hired-in labor $\mathrm{e}\left(\frac{\overleftarrow{H}}{L}\right) =$ Effectiveness of hired labor. $\mathrm{e}' < 0, \mathrm{e}'' > 0, \mathrm{e} < 1$ at $\overleftarrow{H} = 0$ $\mathrm{e}\left(\frac{\overleftarrow{H}}{L}\right) \cdot \overleftarrow{H}$...Effective hired labor supply increases at a decreasing rate with $\overleftarrow{H}, \Rightarrow$ $\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} > 0$ but steadily declining $\Rightarrow 2\,\mathrm{e}' + \mathrm{e}'' \cdot \frac{\overleftarrow{H}}{L} < 0$

Table 1--Partial Derivatives wrt Land Size, *T*, and Outside Wage, *v*

Farmer who also works for hire

	Partials wrt T	Partials wrt v
Tot labor	$\frac{\partial L}{\partial T} = \mathrm{a}_y \cdot \mathrm{f}_T < 0$	$\frac{\partial L}{\partial v} = \mathrm{a}_y \cdot [D - A] + \mathrm{a}_w \quad > 0$
Applied labor	$\frac{\partial A}{\partial T} = -\frac{\mathrm{f}_{TA}}{\mathrm{f}_{AA}} > 0$	$\frac{\partial A}{\partial v} = \frac{1}{\mathrm{f}_{AA}} < 0$
Hired out labor	$\frac{\partial \vec{H}}{\partial T} = \mathrm{a}_y \cdot \mathrm{f}_T + \frac{\mathrm{f}_{TA}}{\mathrm{f}_{AA}} < 0$	$\frac{\partial \vec{H}}{\partial v} = \mathrm{a}_y \cdot [D - A] + \mathrm{a}_w - \frac{1}{\mathrm{f}_{AA}} \quad > 0$
Wage & MP labor	$\frac{\partial w}{\partial T} = \frac{\partial \mathrm{f}_A}{\partial T} = 0$	$\frac{\partial w}{\partial v} = \frac{\partial \mathrm{f}_A}{\partial v} = 1$
MP land & rent	$\frac{\partial \mathrm{f}_T}{\partial T} = \frac{\mathrm{f}_{TT} \cdot \mathrm{f}_{AA} - [\mathrm{f}_{TA}]^2}{\mathrm{f}_{AA}} > 0$ incr rtns	$\frac{\partial \mathrm{f}_T}{\partial v} = \mathrm{f}_{TA} \cdot \frac{\partial A}{\partial v} < 0$

a. Small Farmer Also Works for Hire

Maximize profit:

(17) $$P = \mathrm{f}(T, A) - w \cdot L + v \cdot \vec{H}$$

subject to:

$$\vec{H} + A = L = \mathrm{a}(y, w); \quad \vec{H}, A \geq 0 \tag{18}$$
$$y = P + wD$$

First-order conditions:

$$\begin{aligned} &\vec{H}: \quad [\mathrm{f}_A - v] \cdot \vec{H} = 0; \ \vec{H} \geq 0 \\ &A: \quad [\mathrm{f}_A - w] \cdot A = 0; \quad A \geq 0 \end{aligned} \tag{19}$$

Small farmer also works for hire only when:

$$\mathrm{f}_A = v; \quad \vec{H} > 0 \quad [\mathrm{f}_A - w] \cdot A = 0 \tag{20}$$

The small farmer works for hire only as long as the outside wage equals the marginal product of labor on his own land. The more land the small farmer owns, given a market wage, the more he works on his own land, the less he works for hire, and the less he works in total. This is a pure income effect, since wage is fixed. If wage increases, holding land size constant, wage and income effects pull in opposite directions, but by assumption here, there are no backward-bending labor supply curves. See Table 1.

b. Self-sufficient Farmer

The farmer does not work for hire when:

$$\mathrm{f}_A > v; \quad \vec{H} = 0 \quad [\mathrm{f}_A - w] \cdot L = 0 \tag{21}$$

This is the self-sufficient farmer. The farmer’s wage and marginal product of labor exceed the market wage. So he works only on his own land. The more land he owns, the longer hours he works, the higher his wage and marginal product of labor, and the lower the marginal product of his land. For of course the labor to land ratio falls as land size increases.

	Table 2--Partial derivatives wrt *T* Self-sufficient farmer
Farmer's labor:	$\frac{\partial L}{\partial T} = \frac{\left[a_y \cdot Z + a_w\right] \cdot f_{TA} + a_y \cdot f_T}{J*} > 0$ disregarding income effects (a_y terms). But since L goes to a limit, D, then as T gets large: $\frac{\partial L}{\partial T} \to 0$ so that the ratio of labor to land falls.
Wage& MP labor:	$\frac{\partial w}{\partial T} = \frac{\partial f_A}{\partial T} = \frac{f_{TA} + f_{AA} \cdot a_y \cdot f_T}{J} > 0$
MP land:= rent	$\frac{\partial f_T}{\partial T} = \frac{f_{TT} + f_{TA} \cdot a_y \cdot f_T - \left[a_y \cdot Z + a_w\right] \cdot \left[f_{TT} \cdot f_{AA} - \left[f_{TA}\right]^2\right]}{J} < 0$ except for small T where scale effects may dominate.
AP labor:	$\frac{\partial}{\partial T}\frac{f}{L} = \frac{1}{L^2}\left[f_T \cdot L - \left[f - f_A \cdot L\right] \cdot \frac{\partial L}{\partial T}\right] > 0$ increasing or constant returns
AP land:	$\frac{\partial}{\partial T}\frac{f}{T} = -\frac{1}{T^2}\left[f - f_T \cdot T - f_A\,T \cdot \frac{\partial L}{\partial T}\right] < 0$ except for small T, (scale effects).
	--------- $*J = 1 - \left[a_y \cdot Z + a_w\right] \cdot f_{AA} > 0$

c. Farmer Can Hire Additional Labor:

Assume the effectiveness of hired labor is less than that of the farmer's own labor. Moreover, the effectiveness falls as the ratio of hired to own labor rises, due implicitly to the farmer's increasing difficulty of supervising.

Maximize profit:

(22) $$P = \mathrm{f}(T, A) - w \cdot L - v \cdot \overleftarrow{H}$$

Subject to applied labor is effective hired labor plus owner's labor:

(23) $$A = \mathrm{e}\left(\frac{\overleftarrow{H}}{L}\right) \cdot \overleftarrow{H} + L \qquad \mathrm{e}' < 0, \mathrm{e}'' > 0, \mathrm{e} < 1 \text{ at } \overleftarrow{H} = 0; \text{ etc as above}$$

$$L = \mathrm{a}(y, w); \quad \overleftarrow{H}, A, L \geq 0$$
$$y = P + wD$$

First-order conditions:

(24) $$\overleftarrow{H}: \quad \left[\mathrm{f}_A \left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L}\right] - v\right] \cdot \overleftarrow{H} = 0; \quad \overleftarrow{H} \geq 0$$
$$L: \quad \left[\mathrm{f}_A \cdot \left[1 + \mathrm{e}' \cdot \frac{\overleftarrow{H}^2}{L^2}\right] - w\right] \cdot L = 0; \quad L \geq 0$$

<u>i. Farmer does *not* hire additional labor.</u>

(25) $$\overleftarrow{H}: \quad \mathrm{f}_A \cdot \mathrm{e} - v < 0; \quad \overleftarrow{H} = 0$$
$$L: \quad \mathrm{f}_A - w = 0; \quad L > 0$$

The effective marginal product of hired labor is less than the wage for hired labor, *v*. So the farmer does not hire in labor. If he does not hire out labor either, as in ***b***. above then:

(26) $$v < \mathrm{f}_A = w < \frac{v}{\mathrm{e}} \qquad \overleftarrow{H} = 0; \ \overrightarrow{H} = 0$$

The market wage for hired labor is less than the marginal product of labor on the farmer's land, which equals his wage. However, the marginal product of hired in labor is less than the wage, due to the lower effectiveness of hired than own labor.

ii. Farmer *does* hire additional labor.

$$\text{(27)}\qquad \begin{aligned} &\overleftarrow{H}: &\quad & \mathrm{f}_A \cdot \left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} \right] - v = 0; \quad \overleftarrow{H} > 0 \\ &L: &\quad & \mathrm{f}_A \cdot \left[1 - \mathrm{e}' \cdot \frac{\overleftarrow{H}^2}{L^2} \right] - w = 0; \quad L > 0 \end{aligned}$$

From the assumptions about effectiveness of hired labor:

$$\text{(28)}\qquad \begin{aligned} &0 < \mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} < 1 \\ &1 < 1 - \mathrm{e}' \cdot \frac{\overleftarrow{H}^2}{L^2} \end{aligned}$$

So it follows that:

$$\text{(29)}\qquad v < \frac{v}{\left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} \right]} = \mathrm{f}_A = \frac{w}{\left[1 - \mathrm{e}' \cdot \frac{\overleftarrow{H}^2}{L^2} \right]} < w$$

The marginal product of labor is greater than the wage for hired labor, but less than the farmer's own wage.

Define a new production function for the farmer:

$$\text{(30)}\qquad \overline{\mathrm{f}}(T, L, v) \equiv \mathrm{f}(T, \mathrm{e}\left(\frac{\overleftarrow{H}}{L} \right) \cdot \overleftarrow{H} + L) - v \cdot \overleftarrow{H}$$

Then the equilibrium condition for $\overleftarrow{H}$:

$$\text{(31)}\qquad \mathrm{f}_A \cdot \left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} \right] - v = 0$$

can be rewritten as an equation for the demand for hired labor:

(32) $$\overleftarrow{H} = \mathrm{h}(T, L, v)$$

	Table 3--Partial derivatives of h(*T*,*L*,*v*)
Land:	$\mathrm{h}_T \equiv \frac{\partial \overleftarrow{H}}{\partial T} = \frac{1}{\bar{J}*} \mathrm{f}_{TA} \left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} \right] > 0$
Labor:	$\mathrm{h}_L \equiv \frac{\partial \overleftarrow{H}}{\partial L} = \frac{1}{\bar{J}} \left\{ -\mathrm{f}_A \frac{\overleftarrow{H}}{L^2} \left[2\mathrm{e}' + \mathrm{e}'' \frac{\overleftarrow{H}}{L} \right] + \mathrm{f}_{AA} \left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} \right] \left[1 - \mathrm{e}' \cdot \frac{\overleftarrow{H}^2}{L^2} \right] \right\} < 0$ then >0
Market wage:	$\mathrm{h}_v \equiv \frac{\partial \overleftarrow{H}}{\partial v} = -\frac{1}{\bar{J}} < 0$
	$*\bar{J} = -\left[\mathrm{f}_{AA} \left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} \right]^2 + \frac{\mathrm{f}_A}{L} \left[2\mathrm{e}' + \mathrm{e}'' \frac{\overleftarrow{H}}{L} \right] \right] > 0$

The quantity of hired labor is fully determined by the wage, *v*, the farmer's labor supply, *L*, and the size of his land, *T*. For small quantities of hired labor, the farmer's own labor and hired labor are substitutes so that $\mathrm{h}_L < 0$. For larger quantities of hired labor, they are complements, so that $\mathrm{h}_L > 0$. The new production function $\bar{\mathrm{f}}\,(T,L)$ behaves exactly like the original production function for the farmer who does not hire out or in:

	Table 4--Partial derivatives with respect to land size, *T*. **Farmer who hires additional labor**.
Owner's labor:	$\frac{\partial L}{\partial T} = \frac{\left[\mathrm{a}_y \cdot Z + \mathrm{a}_w\right] \cdot \bar{\mathrm{f}}_{TL} + \mathrm{a}_y \cdot \bar{\mathrm{f}}_T}{J\,*} > 0$
Hired labor:	$\frac{\partial \overleftarrow{H}}{\partial T} = \mathrm{h}_L \frac{\partial L}{\partial T} + \frac{\mathrm{h}_T}{J} > 0$
Applied labor:	$\frac{\partial A}{\partial T} = \frac{1}{\mathrm{f}_A}\left[v \frac{\partial \overleftarrow{H}}{\partial T} + w \frac{\partial L}{\partial T}\right] > 0$
	$\frac{\partial \overleftarrow{H}}{\partial T}, \frac{\partial A}{\partial T} \to 0$ as T gets large, by assumption about supervision fn e.
Owner's wage:	$\frac{\partial w}{\partial T} = \frac{\partial \bar{\mathrm{f}}_L}{\partial T} = \frac{\bar{\mathrm{f}}_{TL} + \bar{\mathrm{f}}_{LL} \cdot \mathrm{a}_y \cdot \bar{\mathrm{f}}_T}{J} > 0$
MP applied labor:	$\frac{\partial \mathrm{f}_A}{\partial T} = \mathrm{f}_{TA} + \mathrm{f}_{AA} \frac{\partial A}{\partial T} > 0$
MP land for net production:	$\frac{\partial \bar{\mathrm{f}}_T}{\partial T} = \frac{\bar{\mathrm{f}}_{TT} + \bar{\mathrm{f}}_{TL} \cdot \mathrm{a}_y \cdot \bar{\mathrm{f}}_T - \left[\mathrm{a}_y \cdot Z + \mathrm{a}_w\right]\left[\bar{\mathrm{f}}_{TT} \cdot \bar{\mathrm{f}}_{LL} - \left[\bar{\mathrm{f}}_{TL}\right]^2\right]}{J} < 0$ ex for small T (scale effects).
MP land:	$\frac{\partial \mathrm{f}_T}{\partial T} = \mathrm{f}_{TT} + \mathrm{f}_{TA} \cdot \frac{\partial A}{\partial T} < 0$ ex for small T (scale effects).
AP labor:	$\frac{\partial}{\partial T}\frac{\bar{\mathrm{f}}}{L}, \frac{\partial}{\partial T}\frac{\mathrm{f}}{L+H} > 0$ increasing or constant returns
AP land:	$\frac{\partial}{\partial T}\frac{\bar{\mathrm{f}}}{T}, \frac{\partial}{\partial T}\frac{\mathrm{f}}{T} < 0$ except for small T, (scale effects).
	$*J = 1 - \left[\mathrm{a}_y \cdot Z + \mathrm{a}_w\right] \cdot \bar{\mathrm{f}}_{LL} > 0$

	Table 5--Partial derivatives with respect to outside wage, *v*. **Farmer who hires additional labor.**
Owner's labor:	$\frac{\partial L}{\partial v} = \frac{\mathrm{a}_y \cdot \overleftarrow{H} - \mathrm{h}_L \cdot \left[\mathrm{a}_y \cdot Z + \mathrm{a}_w\right] \cdot}{J\,*} > 0$ small $\overleftarrow{H}$, < 0 large $\overleftarrow{H}$.
Hired labor:	$\frac{\partial \overleftarrow{H}}{\partial v} = \frac{\mathrm{h}_L \cdot \left[\mathrm{a}_y \cdot \overleftarrow{H} + \mathrm{a}_y \cdot Z + \mathrm{a}_w\right] + \mathrm{h}_v}{J} < 0$
Applied labor:	$\frac{\partial A}{\partial v} = \frac{1}{\mathrm{f}_A}\left[v\frac{\partial \overleftarrow{H}}{\partial v} + w\frac{\partial L}{\partial v}\right] < 0$
Owner's wage:	$\frac{\partial w}{\partial v} = \frac{\mathrm{a}_y \cdot \overleftarrow{H}\,\bar{\mathrm{f}}_{LL} - \mathrm{h}_L}{J} > 0$ small $\overleftarrow{H}$, < 0 large $\overleftarrow{H}$.
MP labor:	$\frac{\partial \mathrm{f}_A}{\partial v} = \mathrm{f}_{AA}\frac{\partial A}{\partial v} > 0$
	$*J = 1 - \left[\mathrm{a}_y \cdot Z + \mathrm{a}_w\right] \cdot \bar{\mathrm{f}}_{LL} > 0$

An increase in the outside wage leads to less hiring of labor, and an increase in the average product of labor. If the quantity of hired labor is small, the farmer's own labor will increase, to substitute for hired labor. If hired labor is large, the farmer's own labor will decrease.

2. Farmer Can Rent Land In or Out but Not Hire

Notation 5--Rental
$\overleftarrow{V}\ldots$ land rented in as separate parcel.
$\vec{V}\ldots$ land rented out.
$r\ldots$ market rent for land.
$\rho(T,\overleftarrow{V})\ldots$ effective rent to lessee as function of owned land and additional land.
$\sigma(\vec{v},\ell)\ldots$ effective rent to lessor as fn of parcel size $\vec{v}$ and labor per parcel ℓ.
$n\ldots$ number of parcels into which lessor divides his rented land $\vec{V}$.

a. Farmer Can Rent In Land

i. Farmer rents in land with no transactions costs.

Assume no hiring in or out allowed, but a farmer can rent in additional land at a *fixed* rental rate r: Additional land is available as a *separate* parcel of size $\overleftarrow{V}$.

The farmer maximizes profit:

$$\text{(33)} \qquad \text{Max: } P = \mathrm{f}(T, L_T) + \mathrm{f}(\overleftarrow{V}, L_V) - w \cdot \left[L_T + L_V\right] - r \cdot \overleftarrow{V}$$

First-order conditions:

$$\text{(34)} \qquad \begin{array}{lll} \left[\mathrm{f}_{L_T} - w\right] \cdot L_T = 0 & L_T \geq 0 & \\ \left[\mathrm{f}_{L_T} - w\right] \cdot L_V = 0 & L_V \geq 0 & \\ \left[\mathrm{f}_V - r\right] \cdot \overleftarrow{V} = 0 & \overleftarrow{V} > 0; \quad \left[\mathrm{f}_V - r\right] \leq 0 & \overleftarrow{V} = 0 \\ L_T + L_V - \mathrm{a}(P + w \cdot D, w) = 0 & L_T + L_V \geq 0 & \end{array}$$

	Table 6--Partials with respect to *r* and *T*: Rental with no transactions costs.
Rented land, $\overleftarrow{V}$, wrt rent, *r*:	$\frac{\partial \overleftarrow{V}}{\partial r} = \frac{1}{J*}\left[\mathrm{f}_{VL_V}\left[1 - \mathrm{f}_{L_T\,L_T}\left[\mathrm{a}_y Z + \mathrm{a}_w\right]\right] + \mathrm{f}_{L_T\,L_T}\left[1 + \mathrm{a}_y \mathrm{f}_{VL_V} \overleftarrow{V}\right]\right] < 0$ for $J > 0$
Rented land, $\overleftarrow{V}$, wrt own land, *T*:	$\frac{\partial \overleftarrow{V}}{\partial T} = -\frac{1}{J}\left[\mathrm{f}_{VL_V}\left[\mathrm{f}_{TL_T} + \mathrm{a}_y \mathrm{f}_{L_T\,L_T} \mathrm{f}_T\right]\right] < 0$ for $J > 0$
	$*J = \mathrm{f}_{L_T\,L_T} \cdot \mathrm{f}_{VV} - \left[1 - \mathrm{f}_{L_T\,L_T}\left[\mathrm{a}_y Z + \mathrm{a}_w\right]\right]\left[\mathrm{f}^2_{VL_V} - \mathrm{f}_{VV}\mathrm{f}_{L_V\,L_V}\right] > 0$ for no econ scale* $\left[\mathrm{f}^2_{VL_V} - \mathrm{f}_{VV}\mathrm{f}_{L_V\,L_V}\right] > 0$ for econ of scale

1) Demand for rented land, $\overleftarrow{V}$, falls as rental rate, *r*, increases, *except where there are significant economies of scale*, at which point the denominator turns 0 and changes sign, and the equation "blows up." So if, as postulated, there are significant economies of scale at small land size, then there must be a minimum parcel size below which there is no demand.

2) Demand for rented land, $\overleftarrow{V}$, falls as owned land size, *T*, increases. In fact, it can be shown that total land operated, $T+\overleftarrow{V}$, falls as land size increases! This is an income effect in the absence of transaction costs. It flies in the face of the real world fact that debt increases with equity, though not as fast.

ii. Farmer rents in additional land *with* transactions costs.

Assume no hiring in our out allowed, but the farmer can rent additional land at a rental rate, *r times* $\rho(T, \overleftarrow{V})$, where T is owned land, $\overleftarrow{V}$ is the size of a *separate* parcel, and $\rho(T, \overleftarrow{V})$ reflects rental transaction costs. The rental rate is now affected by both the size of the farmer's land and the size of the parcel he proposes to rent. The farmer can control $\overleftarrow{V}$, but not T.

Reasonably, assume that:

$$
(35)\qquad
\begin{aligned}
&\rho(T,\overleftarrow{V}) > 1; \quad \rho(T,\overleftarrow{V}) \to 1, \quad T >> \overleftarrow{V} \\
&\rho_V > 0 \text{ for } \overleftarrow{V} > V_m \text{ a minimum size parcel} \\
&\rho_{VV} = 0 \text{ charge increases linearly with } \overleftarrow{V} \\
&\rho_T < 0 \text{ lower charge for owner of larger parcel} \\
&\rho_{VT} < 0 \text{ less effect of changes in } \overleftarrow{V} \text{ for higher } T
\end{aligned}
$$

Now the farmer maximizes profit with the additional parcel:

$$
(36)\qquad \text{Max: } P \quad = \quad \mathrm{f}(T, L_T) + \mathrm{f}(\overleftarrow{V}, L_V) - w \cdot \left[L_T + L_V\right] - r\rho\left(\overleftarrow{V}, T\right) \cdot \overleftarrow{V}
$$

First-order conditions:

$$
(37)\qquad
\begin{aligned}
&\left[\mathrm{f}_{L_T} - w\right] \cdot L_T \quad = \quad 0 \qquad L_T \geq 0 \\
&\left[\mathrm{f}_{L_V} - w\right] \cdot L_V \quad = \quad 0 \qquad L_V \geq 0 \\
&\left[\mathrm{f}_V - r\rho_V \overleftarrow{V} - \rho r\right] \cdot \overleftarrow{V} \quad = \quad 0 \qquad \overleftarrow{V} \geq 0 \\
&L_T + L_V - \mathrm{a}(P + w \cdot D, w) = 0 \qquad L_T + L_V \geq 0
\end{aligned}
$$

Notice that the marginal product of rented land is now *greater* than the rent:

$$
(38)\qquad \mathrm{f}_V = r\rho_V \overleftarrow{V} + r\rho > r
$$

Table 7--Partial Derivative wrt *T*

Rented land, $\overleftarrow{V}$, wrt owned land, *T*:	$\frac{\partial \overleftarrow{V}}{\partial T} = \frac{1}{J*}\Big\{ f_{VL_V}\Big[-f_{TL_T} + a_y f_{L_T L_T}\big[f_T - r\rho_T \overleftarrow{V} \big]\Big] + \big[r\rho_{VT}\overleftarrow{V} + r\rho_T \big]\Big[f_{L_T L_T}\big[1 - f_{L_V L_V}[a_y Z + a_w]\big] + f_{L_V L_V} \Big]\Big\}.$
	$*J = f_{L_T L_T}[f_{VV} - 2r\rho_V] - \big[1 - f_{L_T L_T}[a_y Z + a_w]\big]\big[f^2_{VL_V} - [f_{VV} - 2\rho r_V] f_{L_V L_V}\big] > 0$ for $\overleftarrow{V} > V_m$

The second half of the expression for $\overleftarrow{V}$ is >0, and can dominate the first half, with the sensible consequence that additional land rented increases with land owned.

b. Farmer Can Rent Out Land

For smaller farmers to rent in land, larger farmers must rent out land. A reasonable assumption about such farmers is that they must spend a minimum amount of time per rented parcel making and supervising the rental contract. The less they supervise per parcel, the lower the rent they can expect to collect. So the supervision cost creates a pressure to rent fewer and larger parcels. On the other hand, the rent that can be collected falls as the size of the rented out parcel increases, for two reasons. First, the large farmer supervises less per acre. Second, larger parcels are rented to relatively larger lessees, who command a lower rent because they are less leveraged.

The large farmer who rents out land must must allocate his land into a portion he operates himself, and a portion he rents out. Either portion can be zero. His division between the two depends upon the technical characteristics of operation versus renting out. As Cleveland has shown elswhere, a higher internal wage and lower internal land cost gives farmers a comparative advantage in activities with greater economies of scale, or lower intrinsic labor intensity, defined as a lower labor share of output at a given wage. Renting out plausibly shows such characteristics. If so, then the quantity, and possibly even the share of land rented out should be higher for larger farmers.

The large farmer maximizes profit by choosing the number of parcels, n, into which to divide the land he rents out, $\vec{V}$. Each parcel has size $\vec{v} = \vec{V}$/n; the farmer spends time ℓ =L/n supervising it.. The rate of rent received is a function of parcel size and supervisory labor:

$$
\begin{aligned}
& r \cdot \sigma(\vec{v}, \ell).. \\
& 0 < \sigma < 1 \\
& \sigma_v < 0; \quad \sigma_{vv} > 0; \quad \sigma_{v\ell} < 0 \\
& \sigma_\ell > 0; \quad \sigma_{\ell\ell} > 0
\end{aligned}
\tag{39}
$$

The farmer maximizes profit:

$$\text{Max:}\quad P = \mathrm{f}(T-\vec{V}, L_T) + nr\sigma\left(\vec{v},\ell\right)\vec{v} - w\left[L_T + n\ell\right]$$

$$= \mathrm{f}(T-\vec{V}, L_T) + \vec{V}r\sigma\left(\frac{\vec{V}}{n},\frac{L_V}{n}\right) - w\left[L_T + L_V\right]$$

(40)

First-order conditions:

(41)

$$\begin{aligned} &\mathrm{f}_{L_T} - w = 0 \quad L_T \geq 0 \\ &\frac{\vec{V}}{n} r\sigma_\ell - w = 0 \quad L_V \geq 0 \\ &\sigma_v \vec{V} + \sigma_\ell L_V = 0 \\ &\mathrm{f}_T - r\left[\sigma + \frac{\vec{V}}{n}\sigma_v\right] = 0 \quad T-\vec{V},\quad \vec{V} > 0;\quad \mathrm{f}_T - r\left[\sigma + \frac{\vec{V}}{n}\sigma_v\right] > 0 \quad V = 0 \end{aligned}$$

Farmer's labor supply equation:

(42) $$L_T + L_V - \mathrm{a}(P + wD, w) = 0$$

Notice that the marginal product of the farmer's land is less than the market rent.

(43) $$\mathrm{f}_T = r\left[\sigma + \frac{\vec{V}}{n}\sigma_v\right] < r\sigma$$

The requirement that effective rent depends on the farmer's level of supervision clearly limits the number of parcels, independent of any economies of scale at small parcel size. Moreover, for any reasonable specification of $\sigma(\vec{v},\ell)$, parcel size rises with size of the farmers total holding, T.

B. The Multi-Period Consumer-Laborer as a Farmer:

Assume a self-sufficient farmer, owning a parcel of size T, (which could be 0). This farmer maximizes present value, instead of profit. When the present value equations are combined with the consumer-laborer equations, the parameters P_i , w_i , and δ_i become endogenous, and W_0 depends on land size.

The farmer maximizes present value instead of profit:

$$\text{Max: } PV = P_0 + \frac{P_1}{1+\delta_1} + \frac{P_2}{[1+\delta_1][1+\delta_2]} + \ldots + \frac{P_i}{\prod_1^i [1+\delta_i]} + \ldots$$
$$= \mathrm{f}\left(T, L_0\right) - w_0 L_0 + \frac{\mathrm{f}\left(T, L_1\right) - w_1 L_1}{1+\delta_1} + \frac{\mathrm{f}\left(T, L_2\right) - w_2 L_2}{[1+\delta_1][1+\delta_2]} + \ldots \qquad (44)$$

which yields the expected first-order conditions:

$$\begin{aligned} &\mathrm{f}_L\left(T, L_0\right) - w_0 = 0 \\ &\mathrm{f}_L\left(T, L_1\right) - w_1 = 0, \text{ etc.} \end{aligned} \qquad (45)$$

So far, since the farmer has no possibility of transforming income from one period to the next, and assuming utility does not change, there can be no saving. Income equals consumption, and all periods are the same:

$$\begin{aligned} PV &= P\frac{1+\delta}{\delta} = [F - wL]\frac{1+\delta}{\delta} \\ W &= [P + wD]\frac{1+\delta}{\delta} = [F + wZ]\frac{1+\delta}{\delta} \end{aligned} \qquad (46)$$

One-period results apply to each period. That is, a larger farmer has a higher marginal product of labor and internal wage, *w*, and a lower marginal product of land. However, with no capital market exchange, it is not possible to compare the internal discount rate, δ, between different size farmers.

Now suppose a small parcel of land $\hat{T}$ comes available. A smaller farmer, with his higher marginal product of land, will clearly offer a higher *rent* for the parcel. However, the parcel owner may turn him down in favor of a larger, less-leveraged lessee at a lower rent. But can the smaller farmer offer a higher *price* for the parcel, and *buy* it outright? If he buys the parcel, he makes an end run around the transactions costs that obstruct rental. What determines the price the smaller farmer can offer?

The farmer maximizes the present value of his farm with the additional parcel, $\hat{T}$, whose price is *p*. Assume that after the purchase in period 0, production stabilizes at the increased level with the additional parcel $\hat{T}$, and the discount rate between periods stabilizes at δ_2, the rate between periods 1 and 2. So the interesting rate is δ_1 , the rate between period 0 and period 1.

The farmer maximizes present value with the additional parcel $\hat{T}$:

(47)

Max: PV

$$= \mathrm{f}\left(T,L_0\right) - w_0 L_0 - p\hat{T} + \frac{\mathrm{f}\left(T,L_{1T}\right) + \mathrm{f}\left(\hat{T},L_{1\hat{T}}\right) - w_1 L_1}{1+\delta_1} + \frac{\mathrm{f}\left(T,L_{2T}\right) + \mathrm{f}\left(\hat{T},L_{2\hat{T}}\right) - w_2 L_2}{\left[1+\delta_1\right]\left[1+\delta_2\right]} + \dots$$

$$= \mathrm{f}\left(T,L_0\right) - w_0 L_0 - p\hat{T} + \frac{\mathrm{f}\left(T,L_{1T}\right) + \mathrm{f}\left(\hat{T},L_{1\hat{T}}\right) - w_1 L_1}{1+\delta_1}\left[1+\frac{1}{\delta_2}\right]$$

The first-order conditions are:

(48)
$$\begin{aligned}
&\mathrm{f}_L\left(T,L_0\right) - w_0 = 0\\
&\mathrm{f}_L\left(T,L_{1T}\right) - w_1 = 0\\
&\mathrm{f}_L\left(\hat{T},L_{1\hat{T}}\right) - w_1 = 0; \quad L_{1\hat{T}} > 0\\
&-p + \frac{\mathrm{f}_T\left(\hat{T},L_{1\hat{T}}\right)}{1+\delta_1}\left[1+\frac{1}{\delta_2}\right] = 0; \quad \hat{T} > 0
\end{aligned}$$

The maximum price, p, that the farmer can pay for an additional parcel of land, $\hat{T}$, thus is directly proportional to the marginal product of the parcel, but also inversely proportional to δ_1, his discount rate between periods 0 and 1.

As shown in the one period models, the smaller the farmer, the higher his marginal product of land, both on his own parcel, and on an additional parcel he obtains. However, his discount rate is proportional to his marginal rate of substitution between periods 0 and 1, which will obviously be higher the larger . $p\hat{T}$. is in proportion to period 0 consumption:

(49)

$$\mathrm{m}(C_0,W_1) = \mathrm{m}(\mathrm{f}\left(T,L_0\right) - w_0 L_0 - p\hat{T}, \left[\mathrm{f}\left(T,L_{1T}\right) + \mathrm{f}\left(\hat{T},L_{1\hat{T}}\right) - w_1 L_1\right]\left[1+\frac{1}{\delta_2}\right]) = 1+\delta_1$$

(50)
$$\begin{aligned}
&\frac{\partial\delta_1}{\partial\hat{T}} = -\mathrm{m}_C\, p + \mathrm{m}_W\ \mathrm{f}_T\left(\hat{T},L_{1\hat{T}}\right)\left[1+\frac{1}{\delta_2}\right] > 0\\
&\frac{\partial\delta_1}{\partial p} = -\mathrm{m}_C \hat{T} > 0
\end{aligned}$$

In fact, since consumption can never go to 0:

(51) $$\delta_1 \to \infty \quad \text{as} \quad \mathrm{f}\left(T, L_0\right) - w_0 L_0 - p\hat{T} \to 0$$

If additional land were available in infinitesmally small plots, so that $p\hat{T}$ did not significantly affect δ_1, then a smaller farmer could always outbid a larger one for an additional bit of land. But the combination of economies of scale and transactions costs compels a minimum parcel size. So the land market fails, and fails most severely for the poorest participants and would-be participants.

III. Effects of Land and Output Taxes

What are the consequences of a land tax, γT, or an output tax, τF?

A. Single Period Effects of Land and Output Taxes

1. Farmer Can Hire In or Out but Not Rent Land

a. Small Farmer Also Works for Hire

Maximize profit:

(52) $$P = \mathrm{f}(T, A)\left[1 - \tau\right] - wL + v\vec{H} - \gamma T$$

subject to:

(53) $$\begin{gathered} \vec{H} + A = L = \mathrm{a}(y, w); \quad \vec{H}, A \geq 0 \\ y = P + wD \end{gathered}$$

First-order conditions:

(54) $$\begin{aligned} &\vec{H}: \quad \left[\mathrm{f}_A\left[1 - \tau\right] - v\right]\vec{H} = 0;\ \vec{H} \geq 0 \\ &A: \quad \left[\mathrm{f}_A\left[1 - \tau\right] - w\right]A = 0; \quad A \geq 0 \end{aligned}$$

The farmer does work for hire if:

(55) $$\mathrm{f}_A\left[1 - \tau\right] = v; \quad \vec{H} > 0 \quad \left[\mathrm{f}_A\left[1 - \tau\right] - w\right] \cdot A = 0$$

All else being equal, an output tax obviously makes hired out labor more attractive. But in equilibrium, the output tax on employers will push in the opposite direction, by lowering the outside wage. Because this model, in isolation, does not include transactions costs, a land tax is neutral except for an income effect. An increase in land tax, compensated by a decrease in output tax to keep income constant, leaves labor supply unchanged, and otherwise has the same marginal effects as a decrease in output tax.

Table 8--Partial Derivatives for Output and Land Taxes

	Partials wrt τ	Partials wrt γ	Compensated γ
Tot labor	$\frac{\partial L}{\partial \tau} = -a_y \cdot f > 0$	$\frac{\partial L}{\partial \gamma} = -a_y T > 0$	$\frac{\partial L}{\partial \gamma_c} = 0; \qquad \frac{\partial \tau}{\partial \gamma_c} = -\frac{T}{f}$
Applied labor	$\frac{\partial A}{\partial \tau} = \frac{f_A}{[1-\tau] f_{AA}} < 0$	$\frac{\partial A}{\partial \gamma} = 0$	$\frac{\partial A}{\partial \gamma_c} = -\frac{T}{f} \frac{f_A}{[1-\tau] f_{AA}} > 0$
Hired out labor	$\frac{\partial \vec{H}}{\partial \tau} = -a_y \cdot f - \frac{f_A}{[1-\tau] f_{AA}} > 0$	$\frac{\partial \vec{H}}{\partial \gamma} = -a_y T = \frac{\partial L}{\partial \gamma} > 0$	$\frac{\partial \vec{H}}{\partial \gamma_c} = \frac{T}{f} \frac{f_A}{[1-\tau] f_{AA}} < 0$
Wage & MP labor	$\frac{\partial w}{\partial \tau} = 0$	$\frac{\partial w}{\partial \gamma} = 0$	
	$\frac{\partial f_A}{\partial \tau} = \frac{f_A}{[1-\tau]} > 0$	$\frac{\partial f_A}{\partial \gamma} = 0$	$\frac{\partial f_A}{\partial \gamma_c} = -\frac{T}{f} \frac{f_A}{[1-\tau]} < 0$
MP land & rent	$\frac{\partial f_T}{\partial \tau} = \frac{f_{TA} f_A}{[1-\tau] f_{AA}} < 0$	$\frac{\partial f_T}{\partial \gamma} = 0$	$\frac{\partial f_T}{\partial \gamma_c} = -\frac{T}{f} \frac{f_{TA} f_A}{[1-\tau] f_{AA}} > 0$

b. Self-sufficient Farmer

Marginal product of labor is too high to justify hiring out:

(56) $$f_A > v; \quad \vec{H} = 0 \quad \left[f_A [1-\tau] - w \right] \cdot L = 0$$

Table 9--Partial Derivatives for Output and Land Taxes

	Partials wrt τ	Partials wrt γ
Farmer's labor:	$\frac{\partial L}{\partial \tau} = -\frac{\left[a_y\, Z + a_w\right]\cdot f_L + a_y f}{J\,*} < 0$	$\frac{\partial L}{\partial \gamma} = -\frac{a_y\, T}{J} > 0$
Farmer's wage:	$\frac{\partial w}{\partial \tau} = -\frac{f_A + f_{AA}\left[1-\tau\right]a_y f}{J} < 0$	$\frac{\partial w}{\partial \gamma} = -\frac{f_{AA}\left[1-\tau\right]a_y\, T}{J} < 0$
MP labor:	$\frac{\partial f_A}{\partial \tau} = f_{AA}\frac{\partial L}{\partial \tau} > 0$	$\frac{\partial f_A}{\partial \gamma} = f_{AA}\frac{\partial L}{\partial \gamma} < 0$
MP land:= rent	$\frac{\partial f_T}{\partial \tau} = f_{TA}\frac{\partial L}{\partial \tau} < 0$	$\frac{\partial f_T}{\partial \gamma} = f_{TA}\frac{\partial L}{\partial \gamma} > 0$
AP labor:	$\frac{\partial}{\partial \tau}\frac{f}{L} = -\frac{\left[f - f_A\, L\right]}{L^2}\frac{\partial L}{\partial \tau} > 0$	$\frac{\partial}{\partial \gamma}\frac{f}{L} = -\frac{\left[f - f_A\, L\right]}{L^2}\frac{\partial L}{\partial \gamma} < 0$
AP land:	$\frac{\partial}{\partial \tau}\frac{f}{T} = \frac{f_A}{T}\frac{\partial L}{\partial \tau} < 0$	$\frac{\partial}{\partial \gamma}\frac{f}{T} = \frac{f_A}{T}\frac{\partial L}{\partial \gamma} > 0$
	$*J = 1-\left[1-\tau\right]\left[a_y \cdot Z + a_w\right]\cdot f_{AA} > 0$	

In this case, transactions costs bar the farmer from hiring labor in or out. The output tax behaves as expected, reducing labor and output. But, remarkably enough, the land tax is no longer neutral. It pushes in the opposite direction from an output tax. It increases labor, production, and output per acre. Given market failure, the income effect produces marginal effects! When the income effect is compensated by a reduction in output tax, the effect of the land tax is reinforced:

Table 10--Compensated Increase in Land Tax	
Compen-sation:	$\frac{\partial \tau}{\partial \gamma_c} = -\frac{1}{\tilde{J}\,*}T\left[1-\left[1-\tau\right]f_{AA}a_w\right] < 0$
Farmer's labor:	$\frac{\partial L}{\partial \gamma_c} = \frac{1}{\tilde{J}}a_w\, T\, f_A > 0$
Farmer's wage:	$\frac{\partial w}{\partial \gamma_c} = \frac{1}{\tilde{J}}T\, f_A > 0$
	$*\tilde{J} = f_A\, Z + f\left[1-\left[1-\tau\right]f_{AA}a_w\right] > 0$

c. Farmer Can Hire Additional Labor

Assume as before the effectiveness of hired labor is less than that of the farmer's own labor. Moreover, the effectiveness falls as the ratio of hired to own labor rises, due implicitly to the farmer's increasing difficulty of supervising.

Maximize profit:

(57) $$P = \mathrm{f}(T, A)[1-\tau] - wL - v\overleftarrow{H} - \gamma T$$

subject to:

(58) $$\begin{aligned} &A = \mathrm{e}\left(\frac{\overleftarrow{H}}{L}\right)\cdot \overleftarrow{H} + L\ ; \quad \mathrm{e}' < 0, \mathrm{e}'' > 0, \mathrm{e} < 1 \text{ at } \overleftarrow{H} = 0;\ \text{etc as above} \\ &L = \mathrm{a}(y, w); \quad \overleftarrow{H}, A, L \geq 0 \\ &y = P + wD \end{aligned}$$

First-order conditions:

(59) $$\begin{aligned} &\overleftarrow{H}: \quad \left[\mathrm{f}_A[1-\tau]\left[\mathrm{e} + \mathrm{e}'\cdot\frac{\overleftarrow{H}}{L}\right] - v\right]\cdot \overleftarrow{H} = 0; \quad \overleftarrow{H} \geq 0 \\ &L: \quad \left[\mathrm{f}_A[1-\tau]\left[1 + \mathrm{e}'\cdot\frac{\overleftarrow{H}^2}{L^2}\right] - w\right]\cdot L = 0; \quad L \geq 0 \end{aligned}$$

<u>i. Farmer does *not* hire additional labor.</u>

(60) $$\begin{aligned} &\overleftarrow{H}: \quad \mathrm{f}_A[1-\tau]\mathrm{e}(0) - v < 0; \quad \overleftarrow{H} = 0 \quad 0 < \mathrm{e}(0) < 1 \\ &L: \quad \mathrm{f}_A[1-\tau] - w = 0; \quad L > 0 \end{aligned}$$

For a given external wage, *v*, the output tax clearly offers a barrier to hiring, so that a farmer must be relatively richer to begin hiring than if the tax were not present.. If he does not hire out labor either, as in **1B**. above then:

(61) $$\frac{v}{[1-\tau]} < \mathrm{f}_A = w < \frac{v}{\mathrm{e}(0)[1-\tau]} \qquad \overleftarrow{H} = 0;\ \overrightarrow{H} = 0$$

The output tax evidently both raises and widens the range of no hiring out or in for a given market wage, though, as will be seen, it also lowers the market wage.

ii. Farmer *does* hire additional labor

The presence of taxes affects the first order conditions:

(62)

$$\overleftarrow{H}: \quad f_A[1-\tau]\left[e + e' \cdot \frac{\overleftarrow{H}}{L}\right] - v = 0; \quad \overleftarrow{H} > 0$$

$$L: \quad f_A[1-\tau]\left[1 - e' \cdot \frac{\overleftarrow{H}^2}{L^2}\right] - w = 0; \quad L > 0$$

And:

(63)

$$v < \frac{v}{\left[e + e' \cdot \frac{\overleftarrow{H}}{L}\right]} = f_A[1-\tau] = \frac{w}{\left[1 - e' \cdot \frac{\overleftarrow{H}^2}{L^2}\right]} < w$$

The farmer's new production function becomes:

(64)

$$\overline{f}(T, L, v, \tau) \equiv f(T, e\left(\frac{\overleftarrow{H}}{L}\right) \cdot \overleftarrow{H} + L)[1-\tau] - v \cdot \overleftarrow{H}$$

And the equilibrium condition for $\overleftarrow{H}$, can be rewritten as an equation for the demand for hired labor:

(65)

$$\overleftarrow{H} = h(T, L, v, \tau)$$

	Table 11--Partial derivatives of h(*T*,*L*,*v*, τ)
Land:	$\mathrm{h}_T \equiv \frac{\partial \overleftarrow{H}}{\partial T} = \frac{1}{\overline{J}*} \mathrm{f}_{TA} \left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} \right] > 0$
Labor:	$\mathrm{h}_L \equiv \frac{\partial \overleftarrow{H}}{\partial L} = \frac{1}{\overline{J}} \left\{ -\mathrm{f}_A \frac{\overleftarrow{H}}{L^2} \left[2\mathrm{e}' + \mathrm{e}'' \frac{\overleftarrow{H}}{L} \right] + \mathrm{f}_{AA} \left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} \right] \left[1 - \mathrm{e}' \cdot \frac{\overleftarrow{H}^2}{L^2} \right] \right\} < 0$ then >0
Market wage:	$\mathrm{h}_v \equiv \frac{\partial \overleftarrow{H}}{\partial v} = -\frac{1}{[1-\tau]\overline{J}} < 0$
Output tax:	$\mathrm{h}_\tau \equiv \frac{\partial \overleftarrow{H}}{\partial \tau} = -\frac{\mathrm{f}}{[1-\tau]\overline{J}} < 0$
	$*\overline{J} = -\left[\mathrm{f}_{AA} \left[\mathrm{e} + \mathrm{e}' \cdot \frac{\overleftarrow{H}}{L} \right]^2 + \frac{\mathrm{f}_A}{L} \left[2\mathrm{e}' + \mathrm{e}'' \frac{\overleftarrow{H}}{L} \right] \right] > 0$

An increased output tax lowers hired labor demand exactly as does an increased external wage. How do output taxes and land taxes affect owner's and hired labor overall, as well as other variables?

Table 12--Partial Derivatives for Output and Land Taxes

	Partials wrt τ	Partials wrt γ
Farmer's labor:	$\frac{\partial L}{\partial \tau} = -\frac{\left[a_y Z + a_w\right] \cdot \bar{f}_L \left[1 + \frac{e + e' \frac{\overleftarrow{H}}{L}}{1 - e' \frac{\overleftarrow{H}^2}{L^2}} h_L\right] + a_y \bar{f}}{J*}$ > 0? small $\overleftarrow{H}$; <0 large $\overleftarrow{H}$	$\frac{\partial L}{\partial \gamma} = -\frac{a_y T}{J} > 0$
Hired labor:	$\frac{\partial \overleftarrow{H}}{\partial \tau} = h_\tau + h_L \frac{\partial L}{\partial \tau} < 0$	$\frac{\partial \overleftarrow{H}}{\partial \gamma} = h_L \frac{\partial L}{\partial \gamma} > 0$
Farmer's wage:	$\frac{\partial w}{\partial \tau} = -\frac{\bar{f}_L \left[1 + \frac{e + e' \frac{\overleftarrow{H}}{L}}{1 - e' \frac{\overleftarrow{H}^2}{L^2}} h_L\right] + \bar{f}_{LL} [1 - \tau] a_y \bar{f}}{J} < 0$	$\frac{\partial w}{\partial \gamma} = -\frac{\bar{f}_{LL}[1 - \tau] a_y T}{J} < 0$
	$*J = 1 - [1 - \tau]\left[a_y \cdot Z + a_w\right] \cdot \bar{f}_{AA} > 0$	

An output tax decreases the quantity of hired labor, while a land tax increases it. And the same consequences follow as for the farmer who does not hire labor: an output tax decreases output per acre, while a land tax increases it, and so forth.

2. Farmer Can Rent Land In or Out but Not Hire

a. Farmer Can Rent In Land

Assume no hiring in our out allowed, but a landowner can rent additional land at a rental rate, r *times* $\rho(T, \overleftarrow{V})$, where T is owned land, $\overleftarrow{V}$ is the size of a *separate* parcel, and $\rho(T, \overleftarrow{V})$ reflects rental transaction costs as specified above.

Output and land taxes have the expected effects on rental of additional land:

$$\text{(66)} \quad \text{Max: } P \quad = \quad \mathrm{f}(T, L_T)[1-\tau] + \mathrm{f}(\overleftarrow{V}, L_V)[1-\tau] - w \cdot [L_T + L_V] - r \cdot \rho\left(\overleftarrow{V}, T\right) \cdot \overleftarrow{V} - \gamma T$$

First-order conditions:

$$\text{(67)} \quad \begin{array}{ll} \left[\mathrm{f}_{L_T}[1-\tau] - w\right] \cdot L_T \;=\; 0 & L_T \geq 0 \\ \left[\mathrm{f}_{L_T}[1-\tau] - w\right] \cdot L_V \;=\; 0 & L_V \geq 0 \\ \left[\mathrm{f}_V[1-\tau] - r\rho_V \overleftarrow{V} - \rho r\right] \cdot \overleftarrow{V} \;=\; 0 & \overleftarrow{V} \geq 0 \\ L_T + L_V - \mathrm{a}(P + wD, w) = 0 & L_T + L_V \geq 0 \end{array}$$

The output tax makes the marginal product of rented land now *even greater* than before in relation to the rent:

$$\text{(68)} \quad \mathrm{f}_V = \frac{r\rho_V \overleftarrow{V} + r\rho}{[1-\tau]} >> r$$

An increase in the output tax reduces the area of land rented in, while an increase in the land tax increases the area of land rented in.

Table 13--Partial Derivatives with respect to Output and Land Taxes	
Rented land, $\overleftarrow{V}$, wrt τ.	$\frac{\partial \overleftarrow{V}}{\partial \tau} = \frac{1}{J^*} \left\{ \begin{array}{l} \mathrm{f}_V \mathrm{f}_{L_V L_V} \left[1 - \mathrm{f}_{L_T L_T} [1-\tau] \left[\mathrm{a}_y Z + \mathrm{a}_w \right] \right] \\ + \mathrm{f}_{L_T L_T} \left[\mathrm{f}_V + \mathrm{f}_{VL_V} [1-\tau] \left[\mathrm{f}_L \left[\mathrm{a}_y Z + \mathrm{a}_w \right] + \mathrm{a}_y \mathrm{f}(T, L_T) \right] \right] \end{array} \right\} < 0$
Rented land, $\overleftarrow{V}$, wrt γ.	$\frac{\partial \overleftarrow{V}}{\partial \gamma} = \frac{1}{J} \mathrm{f}_{L_T L_T} [1-\tau] \mathrm{f}_{VL_V} \mathrm{a}_y T > 0$
	$^*J = [1-\tau] \left[\mathrm{f}_{L_T L_T} [\mathrm{f}_{VV} - 2r\rho_V] - \left[1 - \mathrm{f}_{L_T L_T} [1-\tau] \left[\mathrm{a}_y Z + \mathrm{a}_w \right] \right] \right] \left[\mathrm{f}_{VL_V}^2 - [\mathrm{f}_{VV} - 2\rho r_V] \mathrm{f}_{L_V L_V} \right]$ $> 0 \quad \text{for } \overleftarrow{V} > V_m$

b. Farmer Can Rent Out Land

As for the large landowner who rents *out* land, an output tax reduces the area rented out, while a land tax increases it:

The large landowner maximizes profit by choosing the number of parcels, n, into which to divide the land he rents out, $\vec{V}$. Each parcel has size $\vec{v} = \vec{V}/n$; the landowner spends time $\ell = L/n$ supervising it.. The rate of rent received is a function of parcel size and supervisory labor. The output tax appears to affect only the land, T-$\vec{V}$, that the landowner operates himself, and thus appears to encourage renting out. But as just shown, the output tax also lowers the rent the lessees will pay. In a general equilibrium model, an output tax will in net discourage land rental. The land tax, however, pushes the landowner to work harder, and rent out more land.

$$\text{Max:} \quad P = \mathrm{f}(T - \vec{V}, L_T)[1-\tau] + nr\sigma(\vec{v}, \ell)\vec{v} - w[L_T + n\ell] - \gamma T$$

$$= \mathrm{f}(T - \vec{V}, L_T)[1-\tau] + \vec{V} r\sigma\left(\frac{\vec{V}}{n}, \frac{L_V}{n} \right) - w[L_T + L_V] - \gamma T \qquad (69)$$

B. The Multi-Period Effect of Output and Land Taxes

Return to the farmer bidding for an additional parcel of land, $\hat{T}$, offering a price p. Assume that after the purchase in period 0, production stabilizes at the increased level with the additional parcel $\hat{T}$, and the discount rate between periods stabilizes at δ_2, the rate between periods 1 and 2. Now add in a tax on output at rate τ, and a tax on land at rate γ.

(70)

Max: PV

$$= \mathrm{f}\left(T, L_0\right)\left[1-\tau\right] - w_0 L_0 - p\hat{T} - \gamma T + \frac{\mathrm{f}\left(T, L_{1T}\right)\left[1-\tau\right] + \mathrm{f}\left(\hat{T}, L_{1\hat{T}}\right)\left[1-\tau\right] - w_1 L_1 - \gamma\left[T+\hat{T}\right]}{1+\delta_1}$$

$$+ \frac{\mathrm{f}\left(T, L_{2T}\right)\left[1-\tau\right] + \mathrm{f}\left(\hat{T}, L_{2\hat{T}}\right)\left[1-\tau\right] - w_2 L_2 - \gamma\left[T+\hat{T}\right]}{\left[1+\delta_1\right]\left[1+\delta_2\right]} + \ldots$$

$$= \mathrm{f}\left(T, L_0\right)\left[1-\tau\right] - w_0 L_0 - p\hat{T} + \frac{\mathrm{f}\left(T, L_{1T}\right)\left[1-\tau\right] + \mathrm{f}\left(\hat{T}, L_{1\hat{T}}\right)\left[1-\tau\right] - w_1 L_1 - \gamma\left[T+\hat{T}\right]}{1+\delta_1}\left[1+\frac{1}{\delta_2}\right]$$

The first-order conditions are:

(71)

$$\begin{aligned}
&\mathrm{f}_L\left(T, L_0\right)\left[1-\tau\right] - w_0 = 0 \\
&\mathrm{f}_L\left(T, L_{1T}\right)\left[1-\tau\right] - w_1 = 0 \\
&\mathrm{f}_L\left(\hat{T}, L_{1\hat{T}}\right)\left[1-\tau\right] - w_1 = 0; \quad L_{1\hat{T}} > 0 \\
&-p + \frac{\mathrm{f}_T\left(\hat{T}, L_{1\hat{T}}\right)\left[1-\tau\right] - \gamma}{1+\delta_1}\left[1+\frac{1}{\delta_2}\right] = 0; \quad \hat{T} > 0
\end{aligned}$$

It is obvious from inspection that the output tax and the land tax have opposite effects on the maximum price, p, that the farmer can pay for an additional parcel of land, $\hat{T}$. The output tax cuts into a smaller farmer's advantage in having a higher marginal product of land. The land tax leverages that advantage. In fact, the higher the land tax, the more large farmers are cut out of the bidding altogether, because the land tax exceeds their marginal product of land.

Thus, as in a single period model, an output tax aggravates market failure due to unequal distribution of land ownership, while a land tax counteracts market failure.

Part IV Omitted

(Incorporated into other papers.)

V. Conclusions

These models have many possible extensions:

If land quality varies, it turns out that higher labor costs and lower internal discount rate give larger farmers a comparative advantage in occupying better quality land -- more centrally located, enjoying greater economies of scale. Therefore, data showing regressive use of land understates the effect of inequality to the extent that larger farmers occupy better parcels. Alternatively, a survey that does not control for quality may not show regressive use if land quality varies enough from smaller to larger farmers. On the same quality land, larger farmers have a comparative advantage in less-intensive activities, for example, grazing cattle instead of growing crops. [Cleveland, 1984, Chp. 3]

If farmers can acquire human capital, larger farmers will acquire more--both because they can better afford it, and because it extends their strained personal labor supply and improves their ability to supervise. If social class depends on wealth, as Roemer and Eswaran and Kotwal suggest, wealthier classes obtain more education. [Cleveland, 1984, Chp. 2]

Or suppose other capital is added: appreciating capital like trees, or depreciating capital like buildings. Holding land quality constant, larger farmers will turn over appreciating capital less frequently--let the trees grow longer before harvest. They will keep depreciating capital longer--replace buildings less often, but possibly build them stronger in the first place. [Cleveland, 1984, Chp. 5]

A dynamic version of the numerical general equilibrium model above can generate an endogenous distribution of landownership, which can in turn be used to test the effect of tax policies. To make unequal distribution of landownership a long-run stable dynamic equilibrium turns out to require the assumption that *time preferences* depend on wealth: larger farmers have more future-oriented time preferences. This assumption is completely independent of the prediction that market failure due to supervision costs reduces larger farmers' return on investment--more future-oriented time preferences make the farmers *accept* those lower returns instead of selling off land. [Cleveland, 1984, Chp. 8]

What about risk aversion? Hoff's model assumes insurance market failure together with equal distribution of land ownership and absolute risk aversion. It is not clear how well the model applies to conditions of unequal distribution, where an assumption of relative risk aversion would be more appropropriate, though insurance markets surely fail worst for poorest farmers. Hoff's model also assumes that consumption equals income; her landowners cannot adjust consumption to compensate for insurance market failure. Brian Wright points out that this conventional assumption conflicts with real-life farmers' refusal to purchase crop insurance, even when heavily-subsidized. [Wright, 1993]. As Hoff herself acknowledges, even very poor farmers have substantial ability to protect consumption from income fluctuations. Surveys show that rural households in developing countries can manage their lives so that consumption remains little affected by anticipated fluctuations in agricultural production short of natural catastrophes. [Alderman and Paxon, 1992]

Ultimately, any usefulness of Hoff’s model must depend on whether and to what extent her proposed small output tax helps or hurts the poorest farmers. It is not at all clear that a broad policy of risk mitigation will in net help those who lead the riskiest livesany more than a broad policy of rent control will help the poorest tenants.

Extensions of the models developed above have implications for risk and risk aversion. First, notice that riskiness is a future good (or “bad.”) That means that differences in discount rate affect the value individuals place on avoiding risk. I have shown elsewhere [Cleveland, 1984, Chp. 6], that if the variance of an asset’s income stream increases rapidly into the future, a simple expected value computation makes that asset appear less risky to a high-discount-rate individual than to a low-discount-rate individual. An investment like a crop loan, where (risky) benefits precede liability by a substantial time period, may appear absolutely more valuable to a high-discount-rate individual. If large farmers have a comparative advantage in occupying good quality land, small farmers have a comparative advantage in occupying poor quality land, which is normally riskier land as well. Labor-intensive practices of small farmers, such as mixed and multiple cropping, may mitigate risk. Consequently public policies to diminish the riskiness of farm income in the name of helping small farmers may in fact give an edge to large farmers! Such policies include output taxes.

As for land taxes: The models presented above suggest that, given unequal distribution of land, supervision costs, and economies of scale which require a minimum land parcel size, land and labor markets will fail. Output taxes aggravate the failure as well as diminish market incentives to transfer land from larger, less productive farmers to smaller, more productive farmers. Land taxes push against market failure, pressing large farmers to hire more workers--driving up wages, and to sell land they cannot use more intensively. The best way to help small farmers and landless workers escape their risky situation could be to set land taxes against the market failures that confine them.

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